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Full Fuzzy Parameterized Soft Set Theory with Decision Making

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ABSTRACT

In this present work, we consider the algebraic operations on full fuzzy parameterized soft set and hence study some of its properties. Furthermore, we give application of this concept in solving decision making problem by developing an algorithm, which is demonstrated with a numerical example to validate the theoretical arguments.

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INTRODUCTION

Due to the lack of precision of problems in Economics, Engineering and Environmental Sciences, many theories were developed including: Probability theory, Fuzzy set theory and soft set theory. In 1965, Zadeh developed theory of fuzzy set [53]; where each element of a set is described by its degree of membership. But it has its own difficulty; the inadequacy of parametrization tool associated with the theory as pointed out by Molodstov [32]. To handle this difficulty, Molodstov in [32] introduced the notion of soft set, which served as a mathematical tool in dealing with inadequacy in parametrization. The author also outlined several areas of application of the theory such as: Riemann integration, game theory, operations research, probability theory, etc. As time goes by, lots of works relating to soft set theory has been published, in [29] Maji *et al.* defined some operations on soft sets and used the theory to solve some decision-making problems [30].

In 2005, Chen *et al.* [17] presented a new definition of soft set as an improvement of [30]. Also, scholars like Cagman & Enginoglu introduced and investigated soft matrix theory and applied it to a decision-making problem [13]. In sequel, many researchers in various directions also contributed like Ali *et al.* defined some new operations such as; restricted union, extended intersection, restricted intersection, etc [8]. Furthermore, Herewan & Deris [21], Herawan *et al.* [22], Xiao *et al.* [50, 49], Majumdar & Samanta [31], Acar *et al.* [1], Park *et al.* [37], Sun *et al.* [48], Zhan & Chen [54], etc., studied the algebraic structure of soft set theory.

In a bid to solve more complicated problems, scholars introduced the concept of fuzzy set into the study of soft set theory, leading to the notion of "fuzzy soft set". This of course, has brought about many interesting applications. In this direction, Maji *et al.* are the first contributors to define Fuzzy soft set [28].

Following this definition, many interesting applications of soft set theory have been introduced and developed by researchers. In [43], Roy & Maji investigated some application of fuzzy soft set. Yang *et al.* made some improvement on this concept [51, 52]. Kharal & Ahmad investigated the notion of a mapping on soft classes and fuzzy soft sets, which is of fundamental importance in fuzzy soft set theory, and they studied the properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets [25, 26]. Also, Som introduced fuzzy soft relation on soft set theory [46]. In Cagman *et al.* defined a fuzzy soft set theory and its related properties and fuzzy soft aggregation operator that allows for more efficient way in dealing with decision making problems [14]. Other contributors are [12, 24, 33, 34, 3, 4, 5, 6].

Further development in the direction of combining fuzzy set with soft set theory, Cagman *et al.* introduced the notion of Fuzzy Parameterized Soft Set (*FPS-set*) and gave their related properties [15]. The same authors defined Fuzzy Parameterized Fuzzy Soft Sets; in which the approximate functions are defined from fuzzy parameters set to the fuzzy subsets of universal set [16]. In a related development, Alkhazaleh *et al.* introduced the concept of fuzzy parameterized interval-valued fuzzy soft set and gave its application in decision making [9]. In [2], the authors introduced the concepts of multi-Q-fuzzy parameterized soft set.

Also, in recent years, in addition to applying soft set and fuzzy soft sets to decision making, many authors like Rodriguez *et al.* proposed the comparison score based approach to solving fuzzy soft set based decision making problems [40]. Also, Nasef *et al.* presented another application of soft sets in a decision-making problem for real estate marketing with the help of rough set, and provide an algorithm to select the optimal choice of an object [35]. Again, in [36], the authors applied the notion of fuzzy soft sets in Sanchez's method from [44] [45] for decision making. Furthermore, a way to solve intertemporal choice problems like: savings, investments, spendings, etc., for fuzzy soft sets was developed [7]. Also, the concepts of hesitant fuzzy set and multi fuzzy soft set were combined in order to produce hesitant multi fuzzy soft set. This combination also introduced a new type of level soft set and called root mean square set level soft set (*RMSS-level soft set*), and presented an algorithm and a novel approach to hesitant multi fuzzy soft set based decision making problems by using the *RMSS-level soft set* [18]. Liu *et al.* also developed a method for fuzzy soft set in decision making based on an ideal solution [27].

Again, in recent developments, several contributions have been made by different researchers on the concept of *FPS-set* theory towards solving more realistic decision-making problems. For instance; the work of Rodzi and Ahmad [42] on Fuzzy Parameterized Hesitant Fuzzy Linguistic Term Soft Set (*FPHFLTS-set*) in multi-criteria decision making, which was motivated by studying the work on hesitant fuzzy linguistic term soft set [39] in a fuzzy parameterized environment. The authors also described some related concepts and consider the fundamental operations of *FPHFLTS-set*, and they were able to develop three different algorithms for solving group decision making. Others are: [20, 38, 10, 11, 23, 47, 41].

Apparently, all these works, starting from the example given by Molodstov [32], are choices based on some certain factors, the influence of choice are more to be considered as Fuzzy than Crisp. Therefore, the decision on the selection of any element in the power set of the universal set ($P(U)$) is based on the contribution of each of the parameters. This motivated Edeghagba and Muhammad to introduce the new concept "Full Fuzzy Parameterized Soft Set (*FFP-soft set*)" [19].

Therefore, in this paper, we consider some of the algebraic properties of the operations of the concept of *FFP*-soft set as introduced in [19], upon which some results are verified. Lastly, a fuzzy decision set is defined and the concept is applied to solve a decision making problem by developing new algorithm.

Preliminaries

In this section, we provide some basic definitions following from: [32], [53], [28], [15] and [19].

Definition 2.1. [32] (**Soft Set**): Let U be a universe set, and E be the set of parameters. A pair (F, E) is called a soft set over U if and only if F is a mapping from E into the set of all subsets of the universe set U , i.e., $F: E \rightarrow P(U)$, where $P(U)$ is the power set of U . In other words, Soft set over U is a parameterized family of subsets of U . Every set $F(e)$, for every $e \in E$, from this family may be considered as the set of e -elements of the soft set (F, E) or considered as the set of e -approximate elements of the soft set. Accordingly, we can view a soft set (F, E) as a collection of approximations: $(F, E) = \{F(e) : e \in E\}$.

Definition 2.2. [53] (**Fuzzy Set**): A pair (μ, U) is called a fuzzy set, where U is a non-empty universe set and μ is a function from U into the unit interval $[0, 1]$, (i.e., $\mu: U \rightarrow [0, 1]$) and for all $x \in U$, $\mu(x)$ is referred to as the degree of membership of $x \in U$.

Definition 2.3. [28] (**Fuzzy Soft Set**): Let I^U be the set of all fuzzy sets of U . Then a pair (f, A) is called a fuzzy soft set over U , where A is a subset of the set of parameters E , and f is a mapping from A into I^U . That is, $f: A \rightarrow I^U$, and for each $a \in A$, $f(a) = f_a: U \rightarrow I$, is a fuzzy set on U .

Definition 2.4. [15] (**Fuzzy Parameterized Soft Set**): Let U be an initial universe, E be the set of parameters and A be the fuzzy set over E . A fuzzy parameterized soft set (*FP*-soft set) γ_A on the universe U is defined by the set of ordered pairs:

$$F_A = \{(\mu_A(x)/x, \gamma_A(x)) : x \in E, \gamma_A(x) \in P(U)\}$$

Where $P(U)$ is the power set of U
and the function $\gamma_A: E \rightarrow P(U)$ is
called approximate function such
that $\gamma_A(x) = \emptyset$ if $\mu_A(x) = 0$

Characterized by the membership
function $\mu_A: E \rightarrow [0, 1]$

The value $\mu_A(x)$ of an element x of the
parameters represents its degree of
importance. And it is solely based on the
desirability of the decision maker.

Hence, this means that the approximate
function is defined from fuzzy subset of E to the
crisp subset of the Universe set U .

Definition 2.5. [19] (**FFP-soft set**): Let $A^\sim \subset E^\sim$.
A Full Fuzzy Parameterized Soft Set (*FFP*-soft
set) F_{A^\sim} on the universe U is given as:

$$F_{A^\sim} = \{(\hat{y}, f_{A^\sim}(\hat{y})) : \hat{y} \in A^\sim, f_{A^\sim}(\hat{y}) \in P(U), \mu_{\hat{y}}(x) \in I\}$$

Where E is the set of all parameters, E^\sim is the
set of all possible fuzzy set over E and A^\sim is a
finite subset of E^\sim , $f_{A^\sim}: A^\sim \rightarrow P(U)$ represents the
approximation function of F_{A^\sim} such that:

$f_{A^\sim}(\hat{y}) = \emptyset$ whenever $\mu_{\hat{y}}(x) = 0 \forall x \in E$ and $\hat{y} \in A^\sim$.
Therefore, a fuzzy set \hat{y} over E is as:

$$\hat{y} = (\mu_{\hat{y}}(x_1)/x_1, \mu_{\hat{y}}(x_2)/x_2, \dots, \mu_{\hat{y}}(x_n)/x_n)$$

characterized by the membership function

$$\mu_{\hat{y}}: E \rightarrow [0, 1], E = \{x_1, x_2, \dots, x_n\}.$$

Definition 2.6. [19] (**A^\sim -empty and A^\sim -universal *FFP*-soft set**): Let $F_{A^\sim} \in FFPSS(U, E^\sim)$, where $FFPSS(U, E^\sim)$ is the set of all *FFP*-soft set, then;

1. F_{A^\sim} is called the A^\sim -empty *FFP*-soft set if $F_{A^\sim}(\hat{y}) = \emptyset, \forall \hat{y} \in A^\sim$ and denoted by $F_{\emptyset A^\sim}$. If $A^\sim = \emptyset$, then A^\sim -empty *FFP*-soft

set is called the empty FFP-soft set, denoted by F_\emptyset .

2. F_{A^\sim} is called A^\sim -Universal FFP-soft set if $F_{A^\sim}(y^\wedge) = U \forall y^\wedge \in A^\sim$, denoted by F_{A^\sim} . If $A^\sim = E^\sim$, then A^\sim -Universal FFP-soft set is called the universal FFP-soft set, denoted by F_{E^\sim} .

Definition 2.7. [19] (FFP-soft subset): Let $A^\sim, B^\sim \subseteq E^\sim$ and $F_{A^\sim}, F_{B^\sim} \in \text{FFPSS}(F, E^\sim)$, then F_{A^\sim} is an FFP-soft subset of F_{B^\sim} , denoted by $F_{A^\sim} \subseteq F_{B^\sim}$ if the following condition holds: $\forall y_i \in A^\sim \exists z_j \in B^\sim$ such that $y_i \leq z_j$ and $f_{A^\sim}(y_i) \subseteq f_{B^\sim}(z_j)$.

Definition 2.8. [19] (Complement of an FFP-soft set): Let $F_{A^\sim} \in$

$\text{FFPSS}(U, E^\sim)$. Then the complement of F_{A^\sim} denoted by $F_{A^\sim}^c$ is given as $F_{A^\sim}^c = \{(\hat{y}^c, f_{A^\sim}^c(\hat{y}^c)) : \hat{y}^c \in E^\sim \setminus A^\sim, f_{A^\sim}^c(\hat{y}^c) \in$

Where \hat{y}^c is associated with the membership function $\mu_{\hat{y}^c} : E \rightarrow [0, 1]$ defined by $\mu_{\hat{y}^c}(x) := 1 - \mu_{\hat{y}}(x)$, $\forall x \in E$ and $f_{A^\sim}^c(\hat{y}^c) := \bigcup f_{A^\sim}(y^\wedge) \forall y^\wedge \in A^\sim$

Definition 2.9. [19] (Intersection of two FFP-soft sets): Let $F_{A^\sim}, F_{B^\sim} \in \text{FFPSS}(U, E^\sim)$, then the intersection of F_{A^\sim} and F_{B^\sim} written as $F_{A^\sim} \cap F_{B^\sim}$ is given as

$$F_{A^\sim} \cap F_{B^\sim} = \{(\hat{t}, f_{\hat{C}}(\hat{t})) : \hat{t} \in \hat{C} = \tilde{A} \cap \tilde{B}, f_{\hat{C}}(\hat{t}) \in$$

where $\mu_{\hat{t}}(x) : E \rightarrow [0, 1]$ is the membership function associated with $\hat{t} \in C^\sim$ and defined as:

$$\mu_{\hat{t}}(x) = \mu_{\hat{y}}(x) \wedge \mu_{\hat{z}}(x) = \min\{\mu_{\hat{y}}(x), \mu_{\hat{z}}(x)\}, \forall x \in E, \hat{y} \in A^\sim, \hat{z} \in B^\sim$$

We note that the choice of \hat{y} and \hat{z} are arbitrary.

Definition 2.10. [19] (Set difference of two FFP-soft sets): Let $F_{A^\sim}, F_{B^\sim} \in \text{FFPSS}(U, E^\sim)$. Then, the set difference of F_{A^\sim} and F_{B^\sim} , written as $F_{A^\sim} - F_{B^\sim}$ is given as

$$F_{A^\sim} - F_{B^\sim} = \{(\hat{t}, f_{\hat{C}}(\hat{t})) : \hat{t} \in \tilde{A} - \tilde{B}, f_{\hat{C}}(\hat{t}) \in$$

Where $\mu_{\hat{t}} : E \rightarrow [0, 1]$ is the membership function associated with $\hat{t} \in C^\sim$ and defined as $\mu_{\hat{t}}(x) = \mu_{\hat{y}}(x) \wedge (1 - \mu_{\hat{z}}(x)), \forall x \in E$

Where

$$\mu_{\hat{y}}(x) \wedge (1 - \mu_{\hat{z}}(x)) = \min\{\mu_{\hat{y}}(x), 1 - \mu_{\hat{z}}(x)\} \forall \hat{y} \in A^\sim \text{ and } \hat{z} \in B^\sim$$

$$\text{and } f_{A^\sim}(y^\wedge) - f_{B^\sim}(z^\wedge) = f_{A^\sim}(y^\wedge) \cap f_{B^\sim}^c(z^\wedge)$$

Definition 2.11. [19] (Union of two FFP-soft sets): Let $F_{A^\sim}, F_{B^\sim} \in$

$\text{FFPSS}(U, E^\sim)$, the union of F_{A^\sim} and F_{B^\sim} written as $F_{A^\sim} \cup F_{B^\sim}$ is given as;

$$F_{A^\sim} \cup F_{B^\sim} = \{(\hat{t}, f_{\hat{C}}(\hat{t})) : \hat{t} \in \hat{C} = \tilde{A} \cup \tilde{B}, f_{\hat{C}}(\hat{t}) \in$$

Where A^\sim

$$\cup B^\sim = \{y^\wedge \vee z^\wedge : y^\wedge \in A^\sim, z^\wedge \in B^\sim\}$$

$$\text{and } \hat{y} \vee \hat{z} =$$

$$\max\{\mu_{\hat{y}}(x), \mu_{\hat{z}}(x)\}, \forall x \in E$$

$$\mu_{\hat{y}}(x), \mu_{\hat{z}}(x)\}, \forall x \in E$$

Therefore, for $\hat{t} = y^\wedge \vee z^\wedge \in C^\sim$ with the membership function $\mu_{\hat{t}} : E \rightarrow [0, 1]$, define as $\mu_{\hat{t}}(x) = \max\{\mu_{\hat{y}}(x), \mu_{\hat{z}}(x)\} \forall x \in E$.

3. Algebraic Properties of FFP-soft set Operations

In this section, we consider some of the algebraic properties of FFP-soft sets operations as introduced in [19].

Proposition 3.1. Let $F_{A^\sim}, F_{B^\sim} \in \text{FFPSS}(U, E^\sim)$, then the following hold:

$$1. (F_{A^\sim} \cap F_{B^\sim})^c \subseteq F_{A^\sim}^c \cup F_{B^\sim}^c$$

$$2. (F_{A^\sim} \cup F_{B^\sim})^c \supseteq F_{A^\sim}^c \cap F_{B^\sim}^c$$

Proof. Let $C^\sim = (A^\sim \cap B^\sim)^c$ by Remark 4.12 (from [19]), If $|A^\sim| > |B^\sim|$, then, \exists some

$\hat{y} \in A^\sim$ such that $\exists \hat{z} \in B^\sim$ such that $\hat{t} = \hat{y} \wedge \hat{z}, \hat{t} \in C^\sim$.

Therefore $|C^\sim| = |B^\sim| < |A^\sim|$.

Clearly, $|A^\sim| = |A^{\sim c}|$

and $|B^{\sim c}| = |B^\sim|$. Let

$D^\sim = A^{\sim c} \cup B^{\sim c}$, then

by definition, it

follows that $|D^\sim| =$

$|A^{\sim c}| = |A^\sim|$, then

$|C^\sim| < |A^\sim| = |A^{\sim c}| =$

$|D^\sim|$

Implies $|C^\sim| < |D^\sim|$

Hence,

$$(A^\sim \cap B^\sim)^c : \{\hat{t} \in C^\sim, \hat{y} \in A^\sim, \hat{z} \in B^\sim\}^c$$

$$= \{\mu_{\hat{y}}(x) \wedge \mu_{\hat{z}}(x) : \hat{y} \in A^\sim, \hat{z} \in B^\sim\}^c \forall x \in E$$

$$= \{1 - \min\{\mu_{\hat{y}}(x), \mu_{\hat{z}}(x)\} : \hat{y} \in A^\sim, \hat{z} \in B^\sim\}$$

$$|\tilde{C}| < |\tilde{B}| \subseteq \{\max\{1 - \mu_{\hat{y}}(x), 1 - \mu_{\hat{z}}(x)\} : \hat{y} \in A^\sim, \hat{z} \in B^\sim\}$$

$$= \{\hat{y}^c \wedge \hat{z}^c : \hat{y}^c \in \tilde{A}^c, \hat{z}^c \in \tilde{B}^c\}$$

$$= A^{\sim c} \cup B^{\sim c}$$

Thus, $(A^\sim \cap B^\sim)^c \subseteq (A^{\sim c} \cup B^{\sim c})$.

Likewise, let $f_{\tilde{C}}(\hat{t}) = (f_{\tilde{A}}(\hat{y}) \cap f_{\tilde{B}}(\hat{z}))$

If $|A^\sim| > |B^\sim|$, then

$$|\{f_{A^\sim}(\hat{y}) : \hat{y} \in A^\sim\}| > |\{f_{B^\sim}(\hat{z}) : \hat{z} \in B^\sim\}|$$

So, $\exists \hat{y} \in A^\sim$ such that no $\hat{z} \in B^\sim$ and $\hat{t} \in C^\sim$ such that $\hat{t} = \hat{y} \wedge \hat{z}$

Therefore,

$$|\{f_{B^\sim}(\hat{z}) : \hat{z} \in B^\sim\}| = |\{f_{C^\sim}(\hat{t}) : \hat{t} \in C^\sim\}| <$$

$$|\{f_{A^\sim}(\hat{y}) : \hat{y} \in A^\sim\}|$$

Of course, $\{f_{C^\sim}(\hat{t}) : \hat{t} \in C^\sim\} \subseteq \{f_{B^\sim}(\hat{z}) : \hat{z} \in B^\sim\}$

$\subseteq \{f_{A^\sim}(\hat{y}) : \hat{y} \in A^\sim\}$

By definition, we have that $|\{f_{D^\sim}\}| = |\{f_{A^\sim}\}| > |\{f_{B^\sim}\}|$

Of course $|\{f_{\tilde{D}}\}| \supseteq |\{f_{\tilde{A}^c}\}|$ and $|\{f_{\tilde{B}^c}\}|$
 $|\{f_{\tilde{C}}\}| < |\{f_{\tilde{A}}\}| = |\{f_{\tilde{A}^c}\}| = |\{f_{\tilde{D}}\}|$

Implies $|\{f_{C^\sim}\}| < |\{f_{D^\sim}\}|$

Also, $\{f_{A^\sim} \cap f_{B^\sim}\}^c = \{(f_{A^\sim} \cap f_{B^\sim})^c\}$

$$= \{f_{C^\sim}\} \subseteq \{f_{A^\sim}\}$$

$$= \bigcup \{f_{\tilde{A}^c}\}^c \subseteq \bigcup \{f_{\tilde{D}}\}^c$$

$$\Rightarrow \{f_{D^\sim}\} = \{f_{A^\sim}\}^c \cup \{f_{B^\sim}\}^c$$

Thus, $\{f_{A^\sim} \cap f_{B^\sim}\}^c \subseteq \{f_{A^\sim}\}^c \cup \{f_{B^\sim}\}^c$

□

Remark 3.2. In general, the De Morgans' laws are not satisfied as stated in proposition 3.1 above, but the following corollary gives condition under which it holds.

Corollary 3.3. Let

$$F_{A^\sim}, F_{B^\sim} \in$$

$$FFPSS(U, E^\sim),$$

then:

$$1. (F_{\tilde{A}} \cap F_{\tilde{B}})^c = F_{\tilde{A}^c}^c \cup F_{\tilde{B}^c}^c$$

$$2. (F_{\tilde{A}} \cup F_{\tilde{B}})^c = F_{\tilde{A}^c}^c \cap F_{\tilde{B}^c}^c$$

If and only if $|A^\sim| = |B^\sim|$

Proof. The same as above 3.1. □

Proposition 3.4. (Associative law): Let $F_{A^\sim}, F_{B^\sim}, F_{C^\sim} \in FFPSS(U, E^\sim)$, then the following hold:

$$1. F_{A^\sim} \cap (F_{B^\sim} \cap F_{C^\sim}) = (F_{A^\sim} \cap F_{B^\sim}) \cap F_{C^\sim}$$

$$2. F_{A^\sim} \cup (F_{B^\sim} \cup F_{C^\sim}) = (F_{A^\sim} \cup F_{B^\sim}) \cup F_{C^\sim}$$

Proof. 1. Let $FD^{\sim} = FB^{\sim} \cap FC^{\sim}$, $FG^{\sim} = FA^{\sim} \cap FB^{\sim}$, $FH^{\sim} = FA^{\sim} \cap FD^{\sim}$ and $F_K^{\sim} = F_G^{\sim} \cap F_C^{\sim}$ Such that:

$$y^{\wedge} \in A, \sim z^{\wedge} \in B, \sim t^{\wedge} \in C, \sim d^{\wedge} \in D, \sim g^{\wedge} \in G, \sim h^{\wedge} \in H$$

Therefore,

$$\begin{aligned} F_{\tilde{H}} &= \{(\hat{h}, f_{\tilde{H}}(\hat{h})) : h \in \tilde{A} \wedge \tilde{D}, f_{\tilde{H}}(\hat{h}) = f_{\tilde{A}}(\hat{y}) \\ &= \{((\hat{y} \wedge \hat{d}), f_{\tilde{H}}(\hat{y} \wedge \hat{d})) : \hat{y} \wedge \hat{d} \in \tilde{A} \wedge \tilde{D}, f_{\tilde{H}} \\ &= \{(\hat{y} \wedge (\hat{z} \wedge \hat{t})), f_{\tilde{H}}(\hat{y} \wedge (\hat{z} \wedge \hat{t})) : \hat{y} \wedge (\hat{z} \wedge \hat{t}) \\ &= f_{\tilde{A}}(\hat{y}) \cap f_{\tilde{D}}(\hat{z} \wedge \hat{t}) \\ &= \{((\hat{y} \wedge \hat{z}) \wedge \hat{t}), f_{\tilde{K}}((\hat{y} \wedge \hat{z}) \wedge \hat{t}) : ((\hat{y} \wedge \hat{z}) \wedge \hat{t}) \\ &= f_{\tilde{K}}((\hat{y} \wedge \hat{z}) \wedge \hat{t}) = f_{\tilde{A}}(\hat{y}) \cap f_{\tilde{B}}(\hat{z}) \cap f_{\tilde{D}}(\hat{t}) \\ &= \{(\hat{g} \wedge \hat{t}), f_{\tilde{K}}(\hat{g} \wedge \hat{t}) : \hat{g} \wedge \hat{t} \in \tilde{G} \cap \tilde{C}, f_{\tilde{K}}(\hat{g} \wedge \hat{t}) \\ &= \{(\hat{k}, f_{\tilde{K}}(\hat{k})) : \hat{k} \in \tilde{G} \cap \tilde{C}, f_{\tilde{K}}(\hat{k}) = f_{\tilde{G}}(\hat{g}) \cap f_{\tilde{C}}(\hat{t})\} \\ &= F_{\tilde{K}} \end{aligned}$$

Hence $F_{\tilde{H}} = F_{\tilde{K}}$. This completes part of the proof.

2. The proof is analogous.

□

Proposition 3.5. (Distributive law): Let $F_A^{\sim}, F_B^{\sim}, F_C^{\sim} \in \text{FFPSS}(U, E^{\sim})$, then the following hold:

1. $FA^{\sim} \cap (FB^{\sim} \cup FC^{\sim}) = (FA^{\sim} \cap FB^{\sim}) \cup (FA^{\sim} \cap FC^{\sim})$
2. $FA^{\sim} \cup (FB^{\sim} \cap FC^{\sim}) = (FA^{\sim} \cup FB^{\sim}) \cap (FA^{\sim} \cup FC^{\sim})$

Proof. 1. We will only give this proof since (2) can be proved analogously.

Let $FD^{\sim} = FB^{\sim} \cup FC^{\sim}$, $FE^{\sim} = FA^{\sim} \cap FD^{\sim}$, $FG^{\sim} = FA^{\sim} \cap FB^{\sim}$, $FH^{\sim} = FA^{\sim} \cap FC^{\sim}$ and

$$F_N^{\sim} = F_G^{\sim} \cup F_H^{\sim}$$

Such that:

$$\begin{aligned} a^{\wedge} \in A, \sim b^{\wedge} \in B, \sim c^{\wedge} \in C, \sim d^{\wedge} \in D, \sim e^{\wedge} = a^{\wedge} \\ \wedge d^{\wedge} \in E, \sim g^{\wedge} = a^{\wedge} \wedge b^{\wedge} \in G, \sim h^{\wedge} = a^{\wedge} \wedge c^{\wedge} \in H, \sim n^{\wedge} = g^{\wedge} \vee h^{\wedge} = (a^{\wedge} \wedge b^{\wedge}) \vee (a^{\wedge} \wedge c^{\wedge}) \in N \end{aligned}$$

Therefore

$$\begin{aligned} F_{\tilde{E}} &= \{(\hat{e}, f_{\tilde{E}}(\hat{e})) : e \in \tilde{A} \wedge \tilde{D}, f_{\tilde{E}}(\hat{e}) = f_{\tilde{A}}(\hat{a}) \cap f_{\tilde{D}}(\hat{d}) \\ &= \{((\hat{a} \wedge \hat{d}), f_{\tilde{E}}(\hat{a} \wedge \hat{d})) : \hat{a} \wedge \hat{d} \in \tilde{A} \wedge \tilde{D}, f_{\tilde{E}}(\hat{a} \wedge \hat{d}) \\ &= \{(\hat{a} \wedge (\hat{b} \vee \hat{c})), f_{\tilde{E}}(\hat{a} \wedge (\hat{b} \vee \hat{c})) : \hat{a} \wedge (\hat{b} \vee \hat{c}) \in \tilde{A} \wedge \tilde{D} \\ &= f_{\tilde{A}}(\hat{a}) \cap f_{\tilde{D}}(\hat{b} \vee \hat{c})\} \\ &= \{((\hat{a} \wedge \hat{b}) \vee (\hat{a} \wedge \hat{c})), f_{\tilde{N}}((\hat{a} \wedge \hat{b}) \vee (\hat{a} \wedge \hat{c})) : ((\hat{a} \wedge \hat{b}) \vee (\hat{a} \wedge \hat{c})) \in \tilde{A} \wedge \tilde{D} \\ &= f_{\tilde{N}}((\hat{a} \wedge \hat{b}) \vee (\hat{a} \wedge \hat{c})) = (f_{\tilde{A}}(\hat{a}) \cap f_{\tilde{D}}(\hat{b} \vee \hat{c})) \\ &= (f_{\tilde{A}}(\hat{a}) \cap f_{\tilde{D}}(\hat{b}) \cup f_{\tilde{A}}(\hat{a}) \cap f_{\tilde{D}}(\hat{c})) \\ &= (f_{\tilde{A}}(\hat{a}) \cap f_{\tilde{D}}(\hat{b})) \cup (f_{\tilde{A}}(\hat{a}) \cap f_{\tilde{D}}(\hat{c})) \\ &= (f_{\tilde{G}}(\hat{a} \wedge \hat{b}) \cup f_{\tilde{H}}(\hat{a} \wedge \hat{c})) \\ &= f_{\tilde{N}}(\hat{a} \wedge \hat{b} \vee \hat{a} \wedge \hat{c}) \\ &= f_{\tilde{N}}(\hat{n}) \\ &= F_{\tilde{N}} \end{aligned}$$

Hence $F_{\tilde{E}} = F_{\tilde{N}}$, This completes part of the proof.

2. The proof is analogous.

□

4. FFP-soft set in Decision Making

As an illustration, we are going to use the example presented in [15], with a slight modification;

Example 4.1. Suppose Mr. \tilde{X} wants to purchase a suitable car. Let $A^{\sim} = \{\hat{y}_1, \hat{y}_2, \hat{y}_3, \hat{y}_4\}$ be a chosen subset of E^{\sim} , where the $\hat{y}_{i(s)}$ for $i = 1, 2, 3, 4$ represent four different opinions from four different people that Mr. \tilde{X} consult. Also, let the universal set $U = \{c_1, c_2, c_3, c_4, c_5\}$ which stands for Honda, Toyota, Mercedes, BMW, and

Ford respectively be the set of alternatives to select from, and $E = \{e_1, e_2, e_3, e_4, e_5\}$ which stands for expensive, beautiful, top speed, durability and engine capacity respectively be the set of parameters, specifying the qualities to be considered when making a decision on the car options.

Assume that the four people giving Mr. \tilde{X} their experienced opinion, grade each of the parameter as:

$$\tilde{A} = \left\{ \hat{y}_1 = \left(\frac{0.2}{e_1}, \frac{0.3}{e_2}, \frac{0.4}{e_3}, \frac{0.5}{e_4}, \frac{0.6}{e_5} \right), \hat{y}_2 = \left(\frac{0.3}{e_1}, \frac{0.5}{e_2}, \frac{0.1}{e_3}, \frac{0.2}{e_4}, \frac{0.6}{e_5} \right), \right. \\ \left. \hat{y}_3 = \left(\frac{0.4}{e_1}, \frac{0.2}{e_2}, \frac{0.2}{e_3}, \frac{0.3}{e_4}, \frac{0.5}{e_5} \right), \hat{y}_4 = \left(\frac{0.5}{e_1}, \frac{0.6}{e_2}, \frac{0.5}{e_3}, \frac{0.7}{e_4}, \frac{0.6}{e_5} \right) \right\},$$

from which the following assessments are derived:

$$f_{A^{\sim}}(\hat{y}_1) = \{c_1, c_2\} f_{A^{\sim}}(\hat{y}_2) = \{c_1, c_4, c_5\} f_{A^{\sim}}(\hat{y}_3) = \\ \{c_2, c_3, c_4, c_5\} f_{A^{\sim}}(\hat{y}_4) = \{c_1, c_4\}$$

Therefore, $f_{A^{\sim}}(\hat{y}_1)$ represents the cars which are considered as:
expensive to the degree 0.2 beautiful to the degree 0.3 top
speed to the degree 0.4 durable to the degree 0.5

and the engine capacity is to the degree 0.6

The functional value of $f_{A^{\sim}}(\hat{y}_1)$ is $\{c_1, c_2\}$.

Hence, this indicates the degree by which the first person \hat{y}_1 uses the parameters in E intra-dependently and rate the cars c_1 and c_2 .

Thus, the person fuzzify the set of parameters intra-dependently. For instance, looking at the cost of the cars c_1 and c_2 , before grading the parameter "expensive", the consideration first goes to how beautiful, the top speed, the

engine capacity and how durable the cars are, then the graduation of the parameter "expensive" follows.

So unlike in [15], where the parameters are independently given fuzzy values, here, the fuzziness of each parameter is given intra-dependently so as to make sure that the reason for making choices is clearly defined.

$$\text{So, } f_{A^{\sim}}(\hat{y}) \text{ with } \hat{y} = \left(\mu_{\hat{y}}(e_1)/e_1, \mu_{\hat{y}}(e_2)/e_2, \dots, \mu_{\hat{y}}(e_5)/e_5 \right)$$

is considered to be \hat{y} -approximate elements of $FFPSS(U, E^{\sim})$. Thus $FFPSS(U, E^{\sim})$ gives a description of the cars under consideration based on the valuation of the parameters e_1, e_2, \dots, e_5 , hence dealing with multiple entries.

So from the assessment above:

$$\begin{aligned} f_{\tilde{A}}(y^{\wedge}_1) &= \{c_1, c_2\} \\ f_{\tilde{A}}(y^{\wedge}_2) &= \{c_1, c_4, c_5\} \\ f_{\tilde{A}}(y^{\wedge}_4) &= \{c_1, c_4\} \end{aligned}$$

Thus, the element (car, c_1) will have the valuations:

$(0.2/e_1, 0.3/e_2, 0.4/e_3, 0.5/e_4, 0.6/e_5)$, $(0.3/e_1, 0.5/e_2, 0.1/e_3, 0.2/e_4, 0.6/e_5)$ and $(0.5/e_1, 0.6/e_2, 0.5/e_3, 0.7/e_4, 0.6/e_5)$ respectively.

In this case, to find the \hat{y} – approximate of c_1 , we consider the arithmetic mean \hat{y}_1 , \hat{y}_2 and \hat{y}_4 componentise.

So c_1 approximation is;

$$\begin{aligned} &\left(\frac{0.2 + 0.3 + 0.5}{3}/e_1, \frac{0.3 + 0.5 + 0.6}{3}/e_2, \frac{0.4 + 0.1 + 0.5}{3}/e_3, \right. \\ &\quad \left. \frac{0.5 + 0.2 + 0.7}{3}/e_4, \frac{0.6 + 0.6 + 0.6}{3}/e_5 \right) \\ &= (0.33/e_1, 0.47/e_2, 0.33/e_3, 0.47/e_4, 0.6/e_5). \end{aligned}$$

Thus, $f_{\tilde{A}}(y^{\wedge}) = (0.33/e_1, 0.47/e_2, 0.33/e_3, 0.47/e_4, 0.6/e_5) = \{c_1\}$

a Fuzzy decision set of an FFP-soft set

Here, the idea is to produce a single fuzzy set by combining the approximate functions of soft set, which can be used to evaluate each alternative.

Definition 4.2. Let $F_{\tilde{A}} \in \text{FFPSS}(U, E^-)$, then a fuzzy decision of $F_{\tilde{A}}$, denoted by $F_{\tilde{A}}^d$ is defined by:
 $F_{\tilde{A}}^d = \{\mu_{F_{\tilde{A}}^d}(c)/c : c \in U\}$

which is a fuzzy set over U , it's membership function $\mu_{F_{\tilde{A}}^d} : U \rightarrow [0, 1]$ is defined by:

$$\mu_{F_{\tilde{A}}^d} = \frac{1}{n} \left(\sum_{i=1}^n \left(\frac{1}{\sum_{i=1}^n \chi_{\hat{y}_i}(e_j)} \sum_{e_i \in \hat{y}_j} \mu_{\hat{y}_i}(e_j) \chi_{f_{\tilde{A}}(e_j)}(c_i) \right) \right)$$

where n is the number of parameters, and

$$\chi_{\hat{y}_i}(e_j) = \begin{cases} 1, & \text{if } \hat{y}_i(e_j) \neq 0 \\ 0, & \text{if } \hat{y}_i(e_j) = 0 \end{cases}$$

and $f_{\tilde{A}}(e_j)$ is a crisp subset determined by the parameter e_j and

$$\chi_{f_{\tilde{A}}(e_j)}(c_i) = \begin{cases} 1, & \text{if } c_i \in f_{\tilde{A}}(e_j) \\ 0, & \text{if } c_i \notin f_{\tilde{A}}(e_j) \end{cases}$$

b Algorithm

For the example discussed above, we have the following algorithm in order to come up with the best alternative:

1. Construct a full fuzzy parameterized soft set $F_{\tilde{A}}$ over U
2. Compute the fuzzy decision set
3. Select the largest membership grade $\max \mu_{F_{\tilde{A}}^d}(c)$,

Step 1: The first thing to do for $\text{Mr. } \tilde{A}$ is to evaluate the alternatives according to the assessments of those people he consults and construct a FFP-soft set $F_{\tilde{A}}$. So by using the assessments above, just like we showed how to find the \hat{y} -approximation of c_1 by considering the arithmetic mean of \hat{y}_1, \hat{y}_2 and \hat{y}_4 component wise.

We have from the assessment given; $f_{\tilde{A}}(\hat{y}_1) = \{c_1, c_2\}$

$f_{\tilde{A}}(\hat{y}_2) = \{c_1, c_4, c_5\}$ $f_{\tilde{A}}(\hat{y}_3) = \{c_2, c_3, c_4, c_5\}$ $f_{\tilde{A}}(\hat{y}_4) = \{c_1, c_4\}$

$$F_{\tilde{A}} = \left\{ \left(\left(\frac{0.2}{e_1}, \frac{0.3}{e_2}, \frac{0.4}{e_3}, \frac{0.5}{e_4}, \frac{0.6}{e_5} \right) \{c_1, c_2\} \right), \left(\left(\frac{0.3}{e_1}, \frac{0.5}{e_2}, \frac{0.1}{e_3}, \frac{0.2}{e_4}, \frac{0.6}{e_5} \right) \{c_1, c_4, c_5\} \right), \left(\left(\frac{0.4}{e_1}, \frac{0.2}{e_2}, \frac{0.2}{e_3}, \frac{0.3}{e_4}, \frac{0.5}{e_5} \right) \{c_2, c_3, c_4, c_5\} \right), \left(\left(\frac{0.5}{e_1}, \frac{0.6}{e_2}, \frac{0.5}{e_3}, \frac{0.7}{e_4}, \frac{0.6}{e_5} \right) \{c_1, c_4\} \right) \right\}.$$

Step 2: To find the fuzzy decision set, using the expression:

$$\frac{1}{n} \left(\sum_{i=1}^n \left(\frac{1}{\sum_{i=1}^n \chi_{\hat{y}_i}(e_j)} \sum_{e_i \in \hat{y}_j} \mu_{\hat{y}_i(e_j)} \chi_{f_{\tilde{A}}(e_j)}(c_i) \right) \right)$$

Therefore, for the first alternative c_1 , we have;

$$\begin{aligned} &\Rightarrow \frac{1}{5} \left(\frac{1}{3} ((1 \times 0.2) + (1 \times 0.3) + (0 \times 0.4) + (1 \times 0.5)) + \right. \\ &\quad \frac{1}{3} ((1 \times 0.3) + (1 \times 0.5) + (0 \times 0.2) + (1 \times 0.6)) + \\ &\quad \frac{1}{3} ((1 \times 0.4) + (1 \times 0.1) + (0 \times 0.2) + (1 \times 0.5)) + \\ &\quad \frac{1}{3} ((1 \times 0.5) + (1 \times 0.2) + (0 \times 0.3) + (1 \times 0.7)) + \\ &\quad \left. \frac{1}{3} ((1 \times 0.6) + (1 \times 0.6) + (0 \times 0.5) + (1 \times 0.6)) \right) \\ &\Rightarrow \frac{1}{5} \left(\left(\frac{0.2 + 0.3 + 0.5}{3} \right) + \left(\frac{0.3 + 0.5 + 0.6}{3} \right) + \left(\frac{0.4 + 0.1 + 0.5}{3} \right) + \right. \\ &\quad \left. \left(\frac{0.5 + 0.2 + 0.7}{3} \right) + \left(\frac{0.6 + 0.6 + 0.6}{3} \right) \right) \\ &\Rightarrow \frac{1}{5} (0.33 + 0.47 + 0.33 + 0.47 + 0.6) = 0.441 \end{aligned}$$

\therefore the fuzzy value of the alternative c_1 is 0.44

So, after doing the same for all the alternatives, the fuzzy decision set can be seen as;

$$F_{\tilde{A}}^d = \{0.44/c_1, 0.31/c_2, 0.32/c_3, 0.41/c_4, 0.33/c_5\}$$

Step 3: The final step is to select the largest membership grade, which can be determine by:

$\max \mu_{F_A^d}(c) = 0.44$, which means c_1 is the best alternative, \therefore Honda is the best car to purchase.

5. Conclusion

In this paper, we were able to investigate the set operations of the new approach to fuzzifying the set of parameters of soft set introduced in [19] by examining their algebraic properties. Additionally, we gave the definition of a fuzzy decision set for FFP-soft set and develop a new algorithm. Lastly, an example was given in order to illustrate how the new concept can be applied to solve decision making problem. Our work in this paper is completely theoretical. For further directions, it would be of great interest to extend this result to other set structures.

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