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## Accelerated Failure Time Model with Spatial Dependency and Covariates Interaction Structures: Application to Under-Five Mortality in Nigeria

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### Abstract

Under-five mortality (U5M) remains a pressing public health concern in Nigeria, characterized by persistent regional disparities and influenced by a complex interplay of socioeconomic, demographic, and healthcare-related factors. This study extends the Bayesian Weibull Accelerated Failure Time (AFT) modeling framework by incorporating spatial dependency and covariate interaction structures to better elucidate the determinants of U5M across Nigeria. Utilizing nationally representative data from the 2018 Nigeria Demographic and Health Survey (NDHS), the study evaluates multiple model specifications, including the AFT model with standard covariate interaction structures (SCIS) and logistic-type covariate interaction structures (LTCIS). Spatial clustering at the state level is accounted for using Intrinsic Conditional Autoregressive (ICAR) priors. Model comparison based on Deviance Information Criterion (DIC) and Watanabe-Akaike Information Criterion (WAIC) indicates that the Weibull AFT model with spatial effects and SCIS structure provides the best fit (DIC = 152,058.77; WAIC = 152,049.57), outperforming models without spatial or interaction terms. Key findings highlight breastfeeding as the most protective factor against U5M (time ratio [TR] = 52.405), with its effect further enhanced among children from the richest households (TR up to 3.438). Other significant protective factors include longer birth intervals (24–33 months: TR = 1.346; >33 months: TR = 2.435), maternal education, and higher household wealth. The spatial analysis uncovers distinct geographic disparities, with elevated U5M risks concentrated in several southern states, underscoring the role of regional context. This study advances the methodological and substantive understanding of child mortality by quantifying both spatial heterogeneity and synergistic interactions among key predictors. The findings provide actionable insights for policymakers aiming to design geographically and socioeconomically targeted interventions. Moreover, the research underscores the value of integrating interaction structures within spatial survival models to enhance the precision and relevance of public health analytics.

### Keywords

Accelerated Failure Time Model, Bayesian statistics, Covariate Interaction Structures, Spatial Random Effects, Under-Five Mortality.

## INTRODUCTION

Under-five mortality (U5M) remains a pressing public health challenge in Nigeria, with persistent regional disparities attributed to multidimensional factors such as poverty, inadequate healthcare access, and low maternal education (UNICEF, 2020). To address this, researchers have increasingly adopted advanced survival analysis methods, including Accelerated Failure Time (AFT) models, which are particularly suited for time-to-event data like child survival (Ibrahim et al., 2011).

However, existing studies often oversimplify interactions between determinants—such as socioeconomic status and healthcare utilization despite their complex, context-dependent relationships (Muller & Ulm, 2003). While epidemiological literature interchangeably uses terms like effect modification and statistical interaction (Kleinbaum et al., 1988), the integration of interaction structures into survival models remains underexplored, limiting insights into how combined factors exacerbate or mitigate U5M risks (Li et al., 2019). Addressing these gaps is critical for designing targeted interventions.

Recent studies have employed diverse methodologies to analyze U5M determinants. Spatial survival models, such as those by Daniel (2021) in Kenya and Ghilagaber et al. (2013) in Nigeria, highlight geographic clustering of mortality risks, while Cox proportional hazards models (e.g., Wegbom et al., 2019; Kunnuji et al., 2022) emphasize socioeconomic and demographic predictors like maternal education, wealth index, and birth spacing. In Nigeria, Biradar et al. (2019) and Okoli et al. (2022) further identified regional disparities,

with the North-West exhibiting higher risks linked to poverty and low education. Beyond Nigeria, studies in Sudan (Ahmed et al., 2020), Ethiopia (Yalew et al., 2022), and India (Singh & Singh, 2023) corroborate the role of breastfeeding, birth intervals, and healthcare access, often using frailty or multilevel models to account for unobserved heterogeneity. Notably, Fenta et al. (2025) advanced spatiotemporal analyses across Sub-Saharan Africa, revealing how water access and urbanization interact with spatial effects. Despite these contributions, most studies treat covariates as independent, neglecting synergistic interactions.

A critical limitation in U5M research is the paucity of studies exploring covariate interaction structures within spatially explicit AFT frameworks. While spatial dependencies and temporal trends have been examined (e.g., Fagbamigbe & Nnanatu, 2022; Musa et al., 2020), interactions between key determinants such as how maternal education modifies the effect of healthcare access—are either ignored or reduced to simplistic multiplicative terms (Li et al., 2019). This oversight is particularly salient in AFT models, which, despite their utility for time-to-event data, rarely incorporate interaction terms or nonlinear relationships. For instance, the combined effect of breastfeeding duration and household wealth may follow threshold-based patterns, necessitating flexible modeling approaches. Bridging this gap could unravel context-specific pathways to U5M, informing precision public health strategies. Thus, this study extends spatial AFT models by integrating structured covariate interactions to elucidate the complex drivers of U5M in Nigeria.

## 2.0 Materials and Methods

The study utilized data extracted from the recent Nigeria Demographic and Health Survey (NDHS). Data extraction and cleaning were performed using STATA statistical software, while statistical analysis was conducted using the Integrated Nested Laplace Approximation (INLA), commonly referred to as R-INLA. The outcome variable in this study was the time to death within five years of a child's birth. Children who did not die during this period were considered censored. The event of interest was under-

five mortality (U5M), which was observed if a child died before their fifth birthday.

### 2.1 Variables Classification

The relevance and validity of the study depend on the careful selection of variables from the Nigeria Demographic and Health Survey (NDHS). Only those variables directly linked to the research hypothesis and supported by prior literature on under-five mortality (U5M) determinants were included. This focused approach ensures that the study effectively addresses its objectives and yields meaningful insights into the factors affecting U5M. A detailed description of the selected variables is provided in Table 1.

**Table 1: Description of Variables**

Covariates	Abbreviation	Category and Code
Status	Status	Censored = 0; Dead = 1
Time	Time	Time in months
Maternal Age at Birth	Mage	Age of Mothers at birth in Years
Breastfeeding status	Breastfed	Not Breastfed = 0, Breastfed = 1
Preceding Birth Intervals	PBI	Less than 24 months = 0; 24 - 33 months = 1 More than 33 months = 2
Maternal Education Qualification	MEQ	No Education = 0; primary = 1; Secondary = 2; Higher Education = 3
Wealth Index	WID	Poorest = 0; Poorer = 1; Middle = 2 Richer = 3; Richest = 4
Maternal Body Mass Index	BMI	Underweight = 0, Normal = 1, Overweight/obese = 2
Antenatal Care Utilization	ACU	Incomplete = 0; Complete = 1
Sex of Child	Sex	Male = 0; Female = 1
Type of Place of Residence	TPR	Urban = 0; Rural = 1
Mosquito Net use	MNU	No = 0; Yes = 1
Source of Drinking Water	SoDW	Improved = 0; Unimproved = 1

Source: Author's Compilation

## 2.2 AFT Model Formulation

In the conventional Accelerated Failure Time model, the natural logarithm of the event time,  $\text{Log}(T)$ , is modeled as a linear function of the covariate vector  $M$  (Wei, 1992, Kalbfleisch & Prentice, 2002):

$$\text{Log}(T) = \beta_0 + \sum_{j=1}^p \beta_j M_j + \sigma \varepsilon \quad (1)$$

Taking the exponent of both side of equation (3.1), this may result to:

$$T = \exp(\beta_0 + \sigma \varepsilon) \exp(\sum_{j=1}^p \beta_j M_j) = T_0 \exp(\sum_{j=1}^p \beta_j M_j) \quad (3.2)$$

Where:

$M_j, j = 1, 2, \dots, p$  are the covariates,  $\beta_j, j = 0, 1, \dots, p$  are the regression coefficient,  $\sigma (> 0)$  is the scale parameter and  $\varepsilon$  is a random error which has a specified distribution. The term  $\exp(\sum_{j=1}^p \beta_j M_j)$  is the acceleration factor. Thus, if  $\beta_j > 0$  consequently,  $\exp(\sum_{j=1}^p \beta_j M_j) > 0$ , the covariates  $M_j$  decelerated the survival process and if  $\beta_j < 0$  consequently,  $\exp(\sum_{j=1}^p \beta_j M_j) < 0$ , the covariates  $M_j$  accelerated the survival process.

In order to account for the spatial dependency, the Conventional AFT model (1) has been extended by incorporating spatial random effect (Egbon et al., 2022) as given as below:

$$\text{Log}(T) = \beta_0 + \sum_{j=1}^p \beta_j M_j + w_i + \sigma \varepsilon \quad (3)$$

$$w_i = n_i + v_i$$

Where the frailty term  $w_i$  incorporate the effect of both heterogeneity via the non-spatial frailty  $n_i$  and spatial dependency through the spatial frailty  $v_i$ . The non-spatial frailty  $n_i$  is the random effect which independently and identically distributed with mean 0 and variance  $\sigma_u^2$  that is,  $n_i \sim N(0, \sigma_u^2)$  while the spatial frailty  $v_i$  is define purely based on Intrinsic Conditional

Autoregressive (ICAR) to account for the spatial dependency between clusters (states)

### 2.2.1 Extended Flexible AFT Models

In order to incorporate plausible covariates interactions, the conventional AFT model in (3) was extended to:

$$\text{Log}(T) = \beta_0 + \sum_{j=1}^p \alpha_j M_j + \sum_{k=1}^r \sum_{l>k}^r \vartheta_{kl} (X_{ik} \cdot X_{il}) + w_i + \sigma \varepsilon \quad (4)$$

$\sum_{j=1}^p \alpha_j M_j$  = Linear effects of Covariates  $M_j$  (categorical/continuous)

$\sum_{k=1}^r \sum_{l>k}^r \vartheta_{kl} (X_{ik} \cdot X_{il})$  = Linear effect of the interaction term of plausible covariates

$X_k, X_l$  = Subset of covariates ( $k \neq l$ ) with plausible interactions

$\vartheta_{kl}$  = Regression Coefficients for the interaction terms

Interestingly, the extended AFT models can be implemented in R INLA.

### 2.3 Intrinsic Conditional Autoregressive (ICAR) model.

Spatial dependence is modeled using a conditional autoregressive (CAR) structure in the Besag ICAR prior (Besag et al., 1991). Here, each observation's prior distribution depends on its neighbors, typically following a Gaussian distribution. The mean reflects a weighted average of neighboring values, while the variance captures spatial autocorrelation strength. This approach accounts for spatial dependency through correlated random effects.

Let  $E_{ij} = 1$  if area  $E_i$  and  $E_j$  share a nontrivial border and  $E_{ij} = 0$  otherwise. Set  $E_{ij} = 0$ , then the  $G \times G$  matrix  $E = [E_{ij}]$  is called the adjacency matrix for the region  $D$ . The ICAR prior is defined through the set of all conditional distributions as given below:

$$\pi(ICAR\_Prior) = v_j | \{v_i : i \neq j\} \sim N(\bar{v}_j, \vartheta^2/a_{j+}), j = 1, \dots, G$$

$$(5)$$

Equation 3.23 denoted  $v \sim ICAR(1/\vartheta^2)$ , where  $a_{j+}$  is the number of neighbors of area  $C_j$ ,  $\bar{v}_j = \frac{1}{a_{j+}} \sum_{i:a_{ij}=1} \gamma_i$  is the sample mean of the  $a_{j+}$  values of the neighboring areal unit frailties, and  $\vartheta^2/a_{j+}$  is the conditional variance. In this study, the spatial parameter  $v_i$  is a  $37 \times 1$  vector of spatial effects to account for heterogeneity between states in Nigeria. Let  $w = (v_1, \dots, v_{37})$ .

#### 2.4 Covariate Interaction Structures

The study adopted two covariates interaction structures as used by Muller and Ulm (2003). That is the standard covariate interaction and the logistic transformed covariates structures. According to Muller and Ulm (2003), given two covariates  $M_1$  and  $M_2$ , the standard covariates interaction structure is given by:

$$f(M_1, M_2) = M_1 * M_2$$

$$(6)$$

Here, the covariates are transformed logistic before they are multiplied (Muller & Ulm, 2003). Given two covariates  $M_1$  and  $M_2$ , the logistic transformed covariate structure is express as:

$$f_{tm}(M_1, M_2) = f_{logistic}(M_1) * f_{logistic}(M_2)$$

$$(7)$$

$$f_{logistic}(M) = \frac{1}{1 + \exp(-M)}$$

$$(8)$$

Where  $tm$  denotes transformed before multiplied.

#### 2.5 Weibull Baseline Distribution

The Weibull Distribution also known as type III extreme value distribution is characterized by two parameters. That is, the scale parameter  $\varphi$  and the shape parameter  $\gamma$ .

The cumulative distribution (CDF) for a two parameters Weibull distribution denoted as The probability density function (PDF) of the Weibull distribution denoted as:

$$f(t|\varphi, \gamma) = F'(t: \varphi, \gamma) = \frac{\gamma}{\varphi} \left(\frac{t}{\varphi}\right)^{\gamma-1} \exp\left[-\left(\frac{t}{\varphi}\right)^\gamma\right]$$

$$(9)$$

The survival function of the Weibull distribution is given as:

$$S(t) = 1 - F(t) = \exp\left[-\left(\frac{t}{\varphi}\right)^\gamma\right]$$

$$(10)$$

The hazard function of the Weibull distribution is given as:

$$h(t|\varphi, \gamma) = \frac{\varphi}{\gamma} \left(\frac{t}{\varphi}\right)^{\varphi-1}$$

$$(11)$$

#### 2.6 Bayesian Inference

A typical Bayesian workflow includes three major steps that is the prior distribution, Likelihood Function  $L(\theta)$  and Posterior distribution  $\pi(\theta|D)$ .

##### Step I: Prior Distribution for Parameters and Regression Coefficient

This study employs a non-informative independent gamma distribution as the prior for baseline parameters. Gamma distributions offer flexibility, accommodating both non-informative (uniform) priors and marginal priors for regression coefficients. Similar priors have been widely adopted in prior literature, including studies by Alvares et al. (2021), Muse et al. (2022a, 2022b), and Khan & Basharat (2022). Here,

$$\pi(\varphi) \sim G(c_1, d_1) = \frac{d_1^{c_1}}{\Gamma(c_1)} \varphi^{c_1-1} e^{-d_1\varphi}; c_1, d_1, \varphi > 0$$

$$(12)$$

$$\pi(\gamma) \sim G(c_2, d_2) = \frac{d_2^{c_2}}{\Gamma(c_2)} \gamma^{c_2-1} e^{-d_2\gamma}; c_2, d_2, \gamma > 0$$

$$(13)$$

The hyperparameters of the prior distributions can be easily estimated using historical data from the baseline distribution

(Muse et al., 2022b). Assuming a normal prior distribution for the regression coefficients, we have:

$$\pi(\beta) \sim G(c_3, d_3), \tag{14}$$

$$\beta = \alpha, \vartheta$$

The prior distribution for the spatial parameter  $w_i$  in this study is a  $37 \times 1$  vector of spatial random effects to account for heterogeneity between states in Nigeria. Let  $w = (w_1, \dots, w_{37})$ . The spatial parameter  $w$  was modelled via Baseg-ICAR prior, that is:

$$\pi(w) \sim ICAR \tag{15}$$

The joint prior distribution of all unknown parameters is given by:

$$\pi(\varphi, \gamma, \beta, w) = \pi(\varphi)\pi(\gamma)\pi(\beta)\pi(w) \tag{16}$$

**Likelihood Function**

Let assumed that subjects are observed at  $g$  distinct spatial locations  $g_1, \dots, g_k$ . Let also assumed that  $t_{ij}$  be the (possibly censored) survival time, for subject  $j$  at location  $g_i$  and  $M_{ij}$  be the corresponding p-dimensional vector of covariates,  $i = 1, \dots, k, j = 1, \dots, n_i$ ; let  $n = \sum_{i=1}^k n_i$ .

The likelihood function is given by:

$$L(\theta|T, \delta) = \prod_{i=1}^n \left[ \frac{\varphi}{t_{ij}} \left( \frac{t_{ij}}{\gamma} \right)^\varphi e^{\eta_i} \exp \left( - \left( \frac{t_{ij}}{\gamma} \right)^\varphi e^{\eta_i} \right) \right]^{\delta_{ij}} \times \left[ \exp \left( - \left( \frac{t_{ij}}{\gamma} \right)^\varphi e^{\eta_i} \right) \right]^{1-\delta_{ij}} \tag{12}$$

Where:

$$\eta_i = \beta_0 + \sum_{j=1}^p \beta_j M_j + \sum_{k=1}^r \sum_{l>k}^r \vartheta_{kl} (X_{ik} \cdot X_{il}) + w_i$$

$\varphi$  = is the Weibull shape parameter

$\gamma$  = is the Weibull scale parameter

$\delta_{ij}$  = is the event indicator (1 for death, 0 for censored)

$t_{ij}$  = is the survival time

**Step III: Posterior distribution  $\pi(\theta|D)$**

Using Bayes' Theorem, the joint posterior density function is expressed as the multiplication of the likelihood function and the prior distribution as given below:

$$p(\theta|T) = L(\theta|T, \delta) \times \pi(\varphi, \gamma, \beta, w) \tag{3.39}$$

The joint posterior density was analytically intractable due to integration challenges, so inference was performed using Integrated Nested Laplace Approximation (Martino & Riebler, 2014). The parameters of interest were estimated using R-INLA in R (R Core Team, 2019), which computes posterior marginal distributions for statistical inference.

**3.0 Results**

**Table 2: Model Comparison**

Models	DIC	WAIC
Weibull AFT Model	152397.19	152393.06
Weibull AFT with Spatial Random Effect	152355.10	152345.08
Weibull AFT with SCIS	152100.58	152096.50
Weibull AFT with Spatial Random Effect and SCIS*	152058.77	152049.57
Weibull AFT with LTCIS	152134.31	152130.53
Weibull AFT with Spatial Random Effect and LTCIS	152093.53	152084.61

Source: Authors Compilation

Note: \*Indicates the best fitted model.

SCIS = Standard Covariate Interaction Structure

LTCIS = Logistic Covariate Interaction Structure

The model comparison results in Table 2 demonstrate that the Weibull AFT model with both spatial random effects and standard covariate interaction structures (SCIS) provides the best fit for under-five mortality data, as evidenced by its lowest Deviance Information Criterion (DIC = 152,058.77) and Watanabe Information criterion (WAIC = 152,049.57) values. The baseline model without spatial or interaction effects performed worst (DIC: 152,397.19; WAIC: 152,393.06), indicating the importance of accounting for geographic clustering and covariate interactions. Incorporating spatial random effects alone improved model fit (DIC:

152,355.10; WAIC: 152,345.08), while adding SCIS led to further enhancement (DIC: 152,100.58; WAIC: 152,096.50). Models with logistic covariate interaction structures (LTCIS) showed moderate improvement over baseline but were inferior to SCIS models, suggesting that standard multiplicative interactions better capture the relationships between risk factors. These findings highlight that under-five mortality is influenced by both spatial heterogeneity and synergistic effects between covariates. The superior performance of SCIS over LTCIS indicates that simple interaction terms may be more appropriate for this analysis than complex logistic structures.

Table 3: Posterior Mean Estimates of Weibull AFT Model with Spatial Random effect and SCIS

Covariates	Posterior Mean	Time Ratio	Posterior SD.	CI
<b>Maternal Age at Birth</b>	0.002	1.002	0.002	0.002, 0.006
<b>Breastfed</b>	3.959	52.405	0.094	3.775, 4.143
<b>Preceding Birth Intervals</b>				
< 24	<b>Ref</b>			
24 - 33	0.297	1.346	0.082	0.137, 0.458
> 33	0.890	2.435	0.121	0.654, 1.127
<b>Maternal Educ. Qualification</b>				
No Formal Education	<b>Ref</b>			
Primary	0.019	1.019	0.022	-0.024, 0.063
Secondary	0.012	1.012	0.028	0.043, 0.066
Higher Education	0.033	1.033	0.052	0.069, 0.134
<b>Wealth Index</b>				
Poorest	<b>Ref</b>			
Poorer	0.014	1.014	0.041	0.066, 0.093
Middle	0.025	1.025	0.040	0.054, 0.105
Richer	0.029	1.029	0.042	0.052, 0.111
Richest	0.028	1.028	0.043	0.056, 0.112
<b>Maternal Body Mass Index</b>				
Underweight	<b>Ref</b>			
Normal	0.047	1.048	0.039	-0.030, 0.123
Overweight/Obese	0.026	1.026	0.045	-0.062, 0.114
<b>Antenatal Care Utilization</b>				
Incomplete	<b>Ref</b>			
Complete	0.332	1.394	0.072	0.192, 0.473

<b>Sex of Child</b>				
Male	<b>Ref</b>			
Female	-0.031	0.969	0.015	-0.060, 0.002
<b>Type of Place of Residence</b>				
Rural	<b>Ref</b>			
Urban	0.033	1.033	0.021	-0.007, 0.074
<b>Mosquito Net Use</b>				
No	<b>Ref</b>			
Yes	0.020	1.020	0.017	0.014, 0.053
<b>Source of Drinking Water</b>				
Improved	<b>Ref</b>			
Unimproved	-0.011	0.989	0.019	-0.048, 0.027
<b>Mage*Previous Birth Interval</b>				
Mage* < 24 months	<b>Ref</b>			
Mage* 24 - 33 months	0.007	1.007	0.002	0.003, 0.011
Mage* > 33 months	0.019	1.019	0.003	0.013, 0.025
<b>Breastfed* Wealth Index</b>				
Not Breastfed*Poorest	<b>Ref</b>			
Breastfed *Poor	0.503	1.654	0.112	0.282, 0.723
Breastfed *Middle	0.886	2.425	0.105	0.680, 1.092
Breastfed *Rich	1.199	3.317	0.0101	1.001, 1.397
Breastfed *Richer	1.235	3.438	0.101	1.037, 1.432
Spatial Effect Precision	9.56		2.69	9.00, 11.81

Source: Author's Compilation

Note: CI - Credible Intervals

Table 3 presents the posterior estimates results of a Bayesian Weibull AFT model with spatial random effect and SCIS. Time ratio (TR) greater than 1 indicate increase in survival time (decrease in U5M risk); TR less than 1 indicates decrease in survival time (increase in U5M risk) and TR equal to 1 indicates no effect.

The result of the analysis identifies several strong predictors of U5M. Breastfeeding emerges as the most significant protective factor (TR=52.405), with breastfed children showing increase in survival time (decrease in U5M risk). Birth intervals demonstrate clear protective effects where 24-33 months (TR=1.346) and >33 months (TR=2.435)

intervals both significantly reduce mortality risk compared to shorter intervals (< 24 months). Maternal age shows a small but statistically significant positive effect (TR =1.002). Maternal education levels (secondary and higher) were also significant predictor of U5M, secondary (TR = 1.012) and higher education (TR = 1.033). This suggest that children whose mothers had secondary and higher educational level had increase survival time (decrease U5M risk) as compared to the reference category (no formal education).

Wealth index categories show statistically significant protective effects, with progressively increase in survival time in higher wealth quintiles: poorer (TR=1.014),

middle (TR=1.025), richer (TR=1.029), and richest (TR=1.028) compared to the reference category (poorest). Maternal BMI categories show no significant protective associations. Complete antenatal care demonstrates significant benefits (TR=1.394) suggesting that children whose mother had complete antenatal care utilization had increase survival time (decrease U5M risk). Child gender was not a significant predictor of U5M survival time (TR=0.969). Urban residence shows marginally protective effects but not statistically significant (TR=1.033). Also, source of drinking water was not a significant predictor of U5M (TR = 0.989) while mosquito net use (TR=1.020) was a significant predictor

of U5M. The TR = 1.020 > 1 indicates increase in survival time (decrease in U5M risk).

The maternal age and birth interval interaction shows that each year increase in maternal age enhances survival time by 0.7% (TR=1.007) for children with 24-33-month birth intervals and 1.9% (TR=1.019) for those with intervals over 33 months, compared to shorter intervals (<24 months). The breastfeeding and wealth interaction demonstrates that breastfeeding's effect is amplified in wealthier households, increasing survival time by 65.4% (TR=1.654) for poorer, 142.5% (TR=2.425) for middle, 231.7% (TR=3.317) for rich, and 243.8% (TR=3.438) for richest households as compared to the reference category (non-breastfed and poorest).

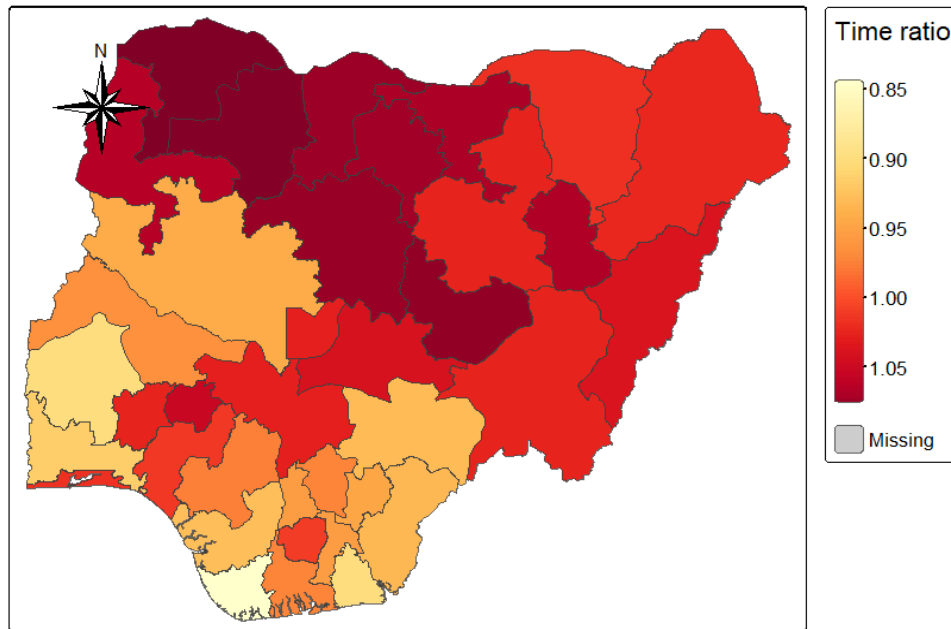


Figure 1: Posterior mean (Average) Spatial Effect of State Based Survival Risk of U5M

The map in figure 1 illustrates geographic variations in U5M risk using time ratios (TR) ranging from 0.85 to 1.05, where TR below 1 indicate decrease in survival time (increase in U5M risk) and TR above 1 suggest low increase in survival time (decrease in U5M risk). Thus, regions with darker red color indicate

increased survival time of about 50% above average. These regions have lower risk of U5M. However, regions with lighter yellow/orange represent regions with decreased survival time of about 15% above average (increase in U5M risk).

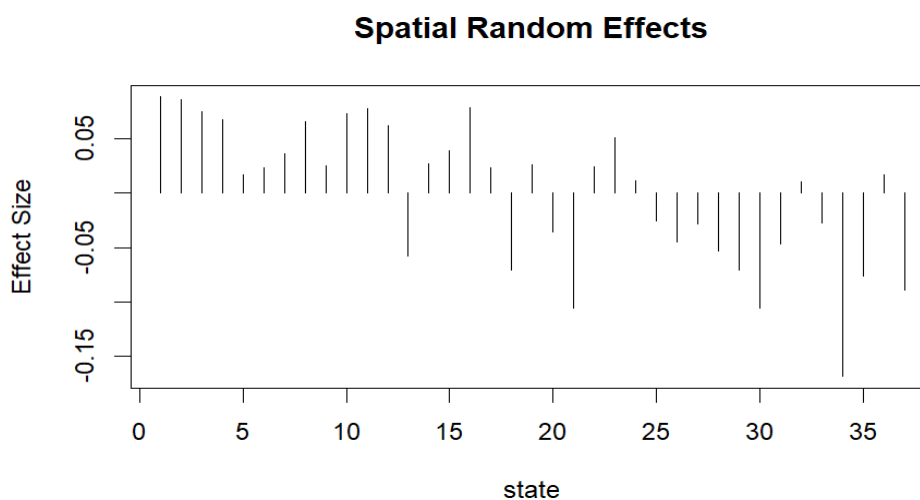


Figure 2: State based Spatial Random Effect Size of U5M

The spatial random effects plot in figure 2 displays effect sizes for all 37 Nigerian states, arranged in numerical order from 1 (Benue) to 37 (Oyo) as captured during the survey. Each vertical line represents a state's random effect, with the y-axis showing the magnitude and direction of the effect size. Most northern states (approximately states 7-20, including Plateau through Zamfara) show predominantly positive effects (above zero), indicating increase survival time (decrease in U5M risk). In contrast, many southern states (approximately states 21-37, including Abia through Oyo) display negative effects (below zero), particularly states like Rivers (#31) and

Osun (#36) which show stronger negative effects around -0.15 indicating decrease in survival time (Increase U5M risk). These spatial random effects represent geographic variations in under-five mortality from the previous map in figure 1) after accounting for other fixed predictors in a statistical model. This result suggest that most Northern states have decrease U5M risk as compared to other regions.

#### 4.0 Discussion of Findings

The findings of this study align with and extend previous research on under-five mortality (U5M) determinants in several

important ways. Like Daniel (2021) in Kenya and Ghilagaber et al. (2013) in Nigeria, our results confirm the significance of geographic clustering in U5M patterns, as evidenced by the spatial variations shown in our mapping analysis. The identification of breastfeeding as the most protective factor (TR=52.405) supports similar findings from Yalew et al. (2022) in Ethiopia and Singh & Singh (2023) in India, while the protective effects of longer birth intervals (24-33 months: TR=1.346; >33 months: TR=2.435) reinforce conclusions from studies across multiple contexts including Ahmed et al. (2020) in Sudan and Wegbom et al. (2019) in Nigeria. The socioeconomic gradients observed in our study, where higher maternal education levels and wealth quintiles progressively reduce U5M risk, mirror patterns documented by Kunnuji et al. (2022) and Okoli et al. (2022) in Nigeria, particularly their findings about the North-West region's vulnerability.

Our analysis advances current understanding by quantifying interaction effects that previous studies often overlooked. The significant interaction between maternal age and birth interval (enhancing survival time by 0.7-1.9% depending on interval length) and the amplified protective effect of breastfeeding in wealthier households (up to 243.8% increased survival time in richest households) provide novel insights into how multiple risk factors operate synergistically. This addresses a critical gap noted in literature reviews, where most studies including Biradar et al. (2019) and Fenta et al. (2025) treated covariates as independent. The spatial patterns we identified through time ratio mapping offer more nuanced geographic targeting opportunities than the broader

regional disparities reported in earlier Nigerian studies, allowing for more precise intervention planning.

The non-significant findings for some variables like maternal BMI and child gender contrast with certain previous studies but align with others, suggesting context-specific variations in U5M determinants. While our results confirm the importance of complete antenatal care (TR=1.394) and mosquito net use (TR=1.020) as protective factors, the marginal effect of urban residence and non-significance of water source differ from some earlier findings, possibly reflecting improvements in rural healthcare access or changes in environmental risk factors over time. These variations underscore the value of our more comprehensive modeling approach that accounts for both individual and interactive effects.

Methodologically, our use of time ratios in the Accelerated Failure Time framework provides more interpretable effect measures than hazard ratios from Cox models used in many previous studies (e.g., Wegbom et al., 2019; Kunnuji et al., 2022). The combination of spatial analysis with interaction modeling bridges two important but typically separate strands of U5M research represented by Ghilagaber et al.'s (2013) spatial work and Fenta et al.'s (2025) interaction analyses. The findings of this current study thus, not only validate several key conclusions from prior research but also extend them through more sophisticated modeling of geographic and covariate interaction effects, offering both methodological and substantive contributions to understanding U5M determinants.

## 5.0 Conclusion

This study makes significant methodological and substantive contributions to understanding under-five mortality in Nigeria through an innovative Accelerated Failure Time model incorporating spatial dependency and covariate interaction structures. The findings of the study demonstrate that the Weibull AFT model with spatial random effects and standard covariate interactions provides the best fit for U5M data, outperforming both simpler models and those with logistic interaction structures. The results reveal that breastfeeding, birth intervals, maternal education, and household wealth are key protective factors, with particularly strong synergistic effects observed between breastfeeding and wealth. Importantly, the spatial analysis identified significant geographic variations in U5M risk across Nigerian states, highlighting the need for regionally targeted interventions. These findings advance current understanding by quantifying complex interaction effects that previous studies often overlooked, while also validating the importance of spatial clustering in U5M patterns.

From a policy perspective, our results suggest that interventions to reduce U5M in Nigeria should adopt a multi-pronged approach that addresses both individual risk factors and their synergistic effects. The amplified protective effect of breastfeeding in wealthier households underscores the importance of combining nutrition programs with poverty reduction strategies. Similarly, the interaction between maternal age and birth intervals suggests that family planning programs should be tailored to mothers' age groups. The spatial mapping of U5M risk

provides valuable guidance for targeting resources to high-risk regions.

## 5.1 Strength of the Study

This study demonstrates several notable strengths that enhance its contribution to the understanding of under-five mortality in Nigeria. The research employs an innovative analytical approach through the Bayesian Weibull accelerated failure time model that effectively incorporates both spatial dependency and covariate interaction structures, providing a more comprehensive framework than traditional survival analysis methods. By systematically examining interaction effects among plausible covariates, the study reveals important synergistic relationships between factors such as breastfeeding practices and household wealth that significantly influence child survival outcomes. The spatial analysis component offers valuable insights into geographic disparities in mortality risk, enabling more precise targeting of interventions. The use of nationally representative demographic and health survey data ensures the findings have strong external validity for policy applications. Furthermore, the application of time ratios for interpreting covariate effects provides more intuitive measures of association compared to conventional hazard ratios. These methodological advancements, combined with the study's rigorous analytical approach, represent significant progress in modeling under-five mortality determinants while offering practical tools for evidence-based child survival interventions in Nigeria and similar settings.

## 5.2 Limitation of the Study

This study has several limitations. Firstly, its reliance on self-reported data from the 2018

Nigeria Demographic and Health Survey may introduce biases, such as recall errors, and fail to fully capture the complexities of under-five mortality rates. Additionally, the model may not account for all potential confounding factors, such as environmental conditions, genetic influences, or healthcare quality, which could impact under-five mortality. Finally, while the study offers valuable insights into the Nigerian context, its findings may not be universally applicable due to Nigeria's unique socio-cultural, economic, and demographic characteristics, limiting their generalizability to other regions or countries.

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