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Deterministic Soft Set with Application in Decision Making

Uduak S. George[1], Edeghagba E. Elijah, Mustapha Ibrahim[2], Yakubu A. Jibrin[2]

Department of Mathematical Sciences, Sa'adu Zungur University, Bauchi State.

Abstract

This paper focuses on the development and application of deterministic soft sets in decision-making processes. Traditionally, a soft set is defined as a mapping from a set of parameters to the crisp subsets of a universal set. Since its introduction by Molodtsov, the theory of soft sets has become a powerful and flexible mathematical tool for handling complex systems characterized by uncertainty, vagueness, and incomplete information. In this study, we propose the concept of *deterministic soft sets* as a modification and extension of the classical soft set framework. The fundamental operations of deterministic soft sets—including complement, union, intersection, AND, and OR—are formally defined and their properties examined. To demonstrate the practical relevance of the proposed model, we present its application in solving decision-making problems. An algorithm based on deterministic soft sets is developed and implemented, providing a systematic approach for decision analysis. The effectiveness of the method is validated through a worked numerical example, which illustrates how the proposed framework can simplify complex decision problems while maintaining logical consistency. The results highlight the potential of deterministic soft sets as an efficient decision-making tool, offering both theoretical insight and practical utility in areas where uncertainty and imprecision play a crucial role.

Keywords
Soft set,
Fuzzy set,
Fuzzy soft set,
Rough set,
Deterministic soft set,
Soft expert set.

Corresponding Author: Uduak S. George, Department of Mathematical Sciences, Sa'adu Zungur University, Bauchi State.
Email: uduaksunday409@gmail.com

INTRODUCTION

Set theory was first introduced to the mathematics world by German Mathematician George Cantor in 1874. Since then, it was popularly incorporated into several fields like economics, environmental sciences, engineering and many more. As a result of its popularity, its limitations to address vague and imprecise real-life problems were discovered. Scientists all over the world made several attempts to address these limitations. One of these attempts was the introduction of fuzzy set by L. A. Zadeh in 1965. Also, in a related direction Molodstov in 1999 developed the theory of soft set to address the uncertainty in parameterization which Zadeh's work failed to address. Molodstov's work, while foundational, several direction and hybride of it have been developed and applied to a lot of areas from algebra to decision making problems. After Molodtsov's work, some operations and application of soft sets were studied in [25] and [40] introduced the concept of soft multisets as a generalization of soft set. The authors in [49] carried out a study on bipolar fuzzy soft sets and Its application in decision-making problems.

Additionally, the study explores three practical applications of the proposed methods in decision-making, with the third application being a real case study on selecting a location for a telecommunication tower. In [40] the authors in their study of soft set theory defined the notions of equal soft set, subset, superset of soft set, compliment of soft set, null soft set, and soft set operations. They offered the first practical application of soft set using the reduction of rough set as mention in [43] and introduced another application of soft set theory in decision making problems for real estate marketing. The authors in [25] proposed parameterization reduction of soft set, and in [36] the authors presented the normal parameterization reduction of soft sets. The authors in [29] proposed a novel approach for normal parameter reduction algorithm of soft set using unit similarity matrix.

Applications of soft set theory in other disciplines and real-life problems are now catching momentum. The authors in [40], gave first practical application of soft sets in decision making problems. It is based on the notion of knowledge of reduction in rough set theory. N-soft and Fuzzy soft sets have also been introduced [19, 39]. In [31] the authors introduced and investigated the notion of full fuzzification of the parameter set. In [13] the authors defined the concepts of possibility fuzzy soft set and fuzzy parameterized interval valued fuzzy soft set and gave their applications in decision making and medical diagnosis. But all the aforementioned models dealt with opinion of one expert at a time. The authors in [24] combined pythagorean fuzzy set and soft expert set and came up with Q-pythagorean fuzzy soft expert set and its application in multi-criteria decision making process.

In line with the fuzzy soft set definition, in [23] the authors provided the fuzzy soft set theory definition along with some related properties. They also developed the fuzzy soft aggregation operator, which enables the creation of more effective decision-making techniques. The same authors in [22] also tried to fuzzify a set of parameters in order to solve more complex problems, and they developed the idea of fuzzy parameterized soft set theory and they also provide properties associated with this theory and suggest a method of decision-making. Furthermore, the authors [2] introduced the notion of Q fuzzy soft sets and studied its operations [3]. Also, they introduced multi-Q fuzzy parameterized soft set and its application [4]. The authors in [38] pointed out that the choice value algorithm which was proposed in [39], is not strongly fit for decision making because all attributes are good descriptions, and the greater the value, the better. However, in practice, attributes may be good, bad or constrained. That is why the algorithm needs to be further improved according to actual problem. So, the authors dedicated to the analysis of the choice based algorithm and the comparison score algorithm and proposed a new decision making algorithm which focus on the application of fuzzy soft set and ideal solutions in decision making problem by laying emphasis on the analysis of decision objectives, attributes analysis and explicit decision function; for which they found that the core of the decision process is the design phase, which is to formulate a model for an identified decision.

Again, in recent years, in addition to applying soft set and fuzzy soft sets to decision making, many authors like [41] presented another application of soft sets in a decision-making problem for real estate marketing with the help of rough mathematics, and provided an algorithm to select the optimal choice of an object. Also, in the same year, In [42] the authors applied the notion of fuzzy soft sets in Sanchez's method [50, 51] of decision making. Furthermore, the authors in [6] developed a way to solve intertemporal choice problems like; savings, investments, spending's, etc., for fuzzy soft sets. Also in 2020, the authors in [27] combined hesitant fuzzy set and multi fuzzy soft set to develop hesitant multi fuzzy soft set and also introduced a new type of level soft set and called it RMSSlevel soft set, and present an algorithm and a novel approach to hesitant multi fuzzy soft set-based decision-making problems by using the RMSS-level soft set. In 2020, the authors in [46] introduced a new notion pertaining to certainty and coverage of a parameter, which is associated with the soft set and they proposed a novel approach that leverages parameter certainty to address decision-making challenges.

In trying to address the parameterized difficulties of instability, in [52] the authors proposed a fuzzy hyper soft set as a hybrid of a fuzzy set and a hyper soft set. Also, the authors in [44] defined partial averages of fuzzy soft sets and provided a new approach for handling a few partial average-based decision-making problems. Rough set theory proposed in [56] presented still another attempt to this problem. Rough set theory, introduced in [56] expresses vagueness, not by means of membership, but employing a boundary region of a set.

The soft expert set is a useful mathematical tool for addressing uncertainty, with significant applications in decision making. The gluing concept of soft set with expert system initiated in [10] to emphasize the due status of the opinions of all experts regarding taking any decision in decision-making system. the authors in [17] proposed neutrosophic vague soft expert set theory. In [20] the authors introduced the concept of possibility fuzzy soft expert set and its properties. The authors in [11] characterized fuzzy soft expert set. and its application. The authors in [20, 21] presented possibility fuzzy soft expert set and fuzzy parameterized soft expert set. In [48] the authors investigated neutrosophic soft expert sets. The aurhors in [8, 9] studied mapping on generalized vague soft expert set and generalized vague soft expert set. The aurhors in [7] explained the application of generalized vague soft expert set in decision making. The aurhors in [32] reviewed Q-neutrosophic soft expert set and its application in decision making.

In [53] the authors studied generalized neutrosophic soft expert set for multiple-criteria decision-making. The authors [14] explained bipolar fuzzy soft expert set and its application in decision making. The aurhors in [15] investigated complex multi-fuzzy soft expert set and its application. The aurhors in [18] presented the com-plex neutrosophic soft expert set and its application in decision making. The aurhors in [45] studied the topsis for single valued neutrosophic soft expert set based multi-attribute decision making problems. In [1] the authors investigated the generalized Q-neutrosophic soft ex-pert set for decision under uncertainty. The aurhors in [5] characterized the multi Q-fuzzy soft expert set and its application. The aurhors in [54] presented the time-neutrosophic soft expert sets and its decision making problem. The aurhors in [16] studied fuzzy parameterised single valued neutrosophic soft expert set theory and its application in decision making. The aurhors in [33] researched generalized fuzzy soft expert set. In [31] the authors presented full fuzzy parameterized soft expert set with application to decision making.

Decision making processes often evaluating multiple alternatives based on various factors, which can significantly influence outcomes across different context [28]. Multi-criteria decision making is a systematic approach used to evaluate and select alternatives based on multiple criteria [35]. It facilitates decision makers in accessing a range of options by comprehensively analyzing trade-offs between criteria such as cost, quality, performance, and risk. Multi-criteria decision making is widely applied to fields like business, engineering, medical science, and disaster management to support complex decision making problems [34].

The majority of these decision making problems involve the complete ranking of decision alternatives rather than selecting the optimal one. For instance, the evaluation of supply chain partners is one of the important of multi-criteria decision making problems that consider different attributes and risk factors to rank the suppliers according to their service level. Evaluation of supply chain partners is crucial for ensuring efficiency and reliabilty of the overall supply chain network [57]. Factors such as supplier performance, delivery time, quality consistency, and cost competitiveness play a significant role in determining their suitability [58].

Table 1: Research Paper (Contribution and Limitation)

Name	Year	Research	Contribution	Limitation
Dmitri Molodtsov	1999	Soft Set	It introduced method of description of elements with their attributes.	It cannot handle Fuzzy evaluation.
Shawkat Alkhazaleh and Abdul Razak Salleh	2011	Soft Expert Set	Introduced multi-expert model.	Not suitable when dealing with fuzzy sets.
Lotfi .A. Zadeh	1965	Fuzzy Set	It can handle vague and imprecise situation.	It is inadequate in parameterization
Naim Çağman, Selman Karataş, and Serdar Enginoglu	2011	Fuzzy Parameterize Soft Set	It introduced fuzzification of set of parameters	Cannot deal with multi-experts situation.
Maruah Bashir and Abdul Razak Salleh	2012	Fuzzy Prameterize Soft Expert Set	solved the problem of multi-experts and fuzzified parameters.	The fuzzification of parameters is partial and static.

Therefore, the introduction of deterministic soft set with application in decision making will be helpful in a real-life problem when face with the dilemma of making choices from multiple alternatives. Deterministic soft set can be helpful in some field of studies like mathematics, economics, medicine etc. It employs external factors which will help in giving a systematic direction in the area of decision making problem. We also defined basic operations on deterministic soft sets, such as complement, union, intersection, OR and AND.

1. Preliminaries

In this section, we provide some basic definitions following from: [26], [43], and [40].

Definition 2.1. [26]. **(Soft set) :** Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subset E$. A pair (F, A) is called a soft set over U, where F is a mapping given by:

$$F : A \longrightarrow P(U)$$

Where, $P(U)$ denotes the power set of U.

Definition 2.2. [43]. **(Soft subset) :** For two soft sets (F, A) and (G, B) over a common universe U, we say that; (F, A) is a soft subset of (G, B) if $A \subset B$, and $F(e) \subseteq G(e)$, for all $e \in A$.

We write, $(F, A) \subseteq (G, B)$,

Definition 2.3. [40]. **(Complement of a soft set) :** The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by: $(F, A)^c = (F^c, \urcorner A)$ where $F^c : \urcorner A \rightarrow P(U)$ is a mapping given by $F^c\ (a) = U - F(\urcorner a)$ for all $a \in \urcorner A$.

Definition 2.4. [40]. **(Equality of a soft set)** Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.5. [40]. **(Not set) :** Not Set of a Set of Parameters. Let $E = \{e_1, e_2, e_3,.., e_n\}$ be a set of parameters. The NOT set of E denoted by $\urcorner E$ is defined by $\urcorner E = (\urcorner e_1, \urcorner e_2, \urcorner e_3,.., \urcorner e_n\}$ where $\urcorner e_i$ = not e_i, $\forall\, i$.

Definition 2.6. [40]. **(Union of a soft set) :.** Union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$, and $\forall\, \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & if \quad \varepsilon \in A - B \\ G(\varepsilon) & if \quad \varepsilon \in B - A \\ F(\varepsilon) \cup\ G(\varepsilon) & if \quad \varepsilon \in A \cup B \end{cases}$$

We write $(F, A)\ \tilde{\cup}\ (G, B) = (H, C)$.

Definition 2.7. [40]. **(Intersection of a soft set)** : Intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cap B$, and $\forall\, \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$, (as both are same set).

We write $(F, A) \cap (G, B) = (H, C)$.

Definition 2.8. [40]. **(Null set) :** A soft set (F, A) over U is said to be a null soft set denoted by Ψ, if $\forall\, \varepsilon \in A, F(\varepsilon) = \emptyset$, (null-set).

Definition 2.9. [40]. **(Absolute set) :** A soft set (F, A) over U is said to be absolute soft set, denoted by $\tilde{A}$, if $\forall\, \epsilon \in A, F(\varepsilon) = U$. Clearly, $\tilde{A}^c = \Phi$, and $\Phi^c = \tilde{A}$

Definition 2.10. [40]. **(AND) :** If (F, A) and (G, B) are two soft sets, then "$(F, A)\ AND\ (G, B)$" denoted by $(F, A) \wedge (G, B)$ is defined by $(F, A) \wedge (G, B) = (H, A \times B)$,

where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall\, (\alpha, \beta) \in A \times B$.

Definition 2.11. [40]. **(OR) :** If (F, A) and (G, B) be two soft sets, then "$(F, A)\ OR\ (G, B)$" denoted by $(F, A) \vee (G, B)$ is defined by $(F, A) \vee (G, B) = (O,\ A \times B)$.

where, $O(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall\, (\alpha, \beta) \in A \times B$.

2. The Concept of Deterministic Soft Set Theory

Definition 3.1 A deterministic soft set over a universe U is a pair $(F, M^{\#}B)$ where F is a mapping given by $F\colon M^{\#}B \rightarrow P(U)$ and $M \subseteq E$ the set of parameters. Here # is a deterministic function $\#\colon B \longrightarrow \{+, -\}$, such that B is the set of deterministic parameters that may influence the set of e-approximate elements of the soft set $(F, M^{\#}B)$. Thus, a deterministic soft set $(F, M^{\#}B)$ over U can be represented by the set of ordered pairs.

$$(F, M^{\#}B) = \left\{(e^{\#}b, F(e^{\#}b)): e \in M, F(e^{\#}b) \in P(U), e^{\#}b \in \{+, -\}\right\}$$

Where F is called approximate function of the deterministic soft set $(F, M^{\#}B)$. It is important to note that the sets $F(e^{\#}b)$ can be arbitrary—some may be empty, while others may have nonempty intersections.

Suppose $U = \{\text{Gold, Hoiuse, Car, Grains}\}$ is a universal set considered to be the set of commoditie, $E = \{\text{Expensive, Beautiful, Valuable, Consumable}\}$ be a set of parameters and $B = \{\text{Desire, Preference, Neccesity}\}$ be a set of deterministic parameters.

Definition 3.2. Complement of a Deterministic Soft Set of a Soft Set

The complement of a deterministic soft set $(F, M^{\#}B)$ is denoted by $(F, M^{\#}B)^c$ and is defined by:

$(F, M^{\#}B)^c = (F^c, \rceil M^{\#}B)$, where $F^c : \rceil M^{\#}B \to P(U)$ is a mapping given by $F^c(\gamma) = U - F(|\gamma)$, $\forall \gamma \in |M^{\#}B$.

Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, M^{\#}B^c)^c = (F, M^{\#}B)$.

Example 3.1.

Suppose the elements of $U, E$and B are represented by $U = \{c_1, c_2, c_3, c_4\}$, $E = \{e_1, e_2, e_3, e_4\}$ and $B = \{b_1, b_2, b_3\}$ respectively.

Assume that;

$F(e_1^{\#}b_1) = \{c_1, c_2, c_3, c_4\}$, $F(e_2^{\#}b_2) = \{c_1, c_3\}$, $F(e_3^{\#}b_2) = \{c_1, c_2, c_4\}$, $F(e_4^{\#}b_3) = \{c_4\}$

Here $(F, M^{\#}B)^c = [(e_1^{\#}b_1)^c = \emptyset$, $(e_2^{\#}b_2)^c = \{c_2, c_4\}$, $(e_3^{\#}b_2)^c = \{c_3\}$, $(e_4^{\#}b_3)^c = \{c_1, c_2, c_3\}]$

Definition 3.3. "AND" Operation on Two Deterministic Soft Set

If $(F, M^{\#}B)$ and $(G, N^{\#}B)$ are two deterministic soft sets then "$(F, M^{\#}B)\ AND\ (G, N^{\#}B)$" denoted by $(F, M^{\#}B) \wedge (G, N^{\#}B)$ is defined by:

$(F, M^{\#}B) \wedge (G, N^{\#}B) = (H_0, M^{\#}B \wedge N^{\#}B)$ where $H_0(\gamma, \mu) = F(\gamma) \cap G(\mu), \forall (\gamma, \mu) \in M^{\#}B \wedge N^{\#}B$.

Example 3.2. Suppose $U = \{\text{Gold, House, Car, Grains, Diamond}\}$ is a universal set, consider the deterministic soft set $(F, M^{\#}B)$ where $M = E = \{\text{Expensive, Beautiful, Valuable,}\}$ be a set of parameters and $B = \{\text{Desire, Preference, Necessity}\}$ be a set of deterministic parameters and $(G, N^{\#}B)$ where $N = E = \{Expensive, Beautiful, Consumable\}$ be a set of parameters and $B = \{Desire, Preference, Necessity\}$.

Suppose the elements of $U, E$and B are represented by $U = \{c_1, c_2, c_3, c_4, c_5\}$, $M = E = \{e_1, e_2, e_3\}$, $N = E = \{e_1, e_2, e_3\}$ and $B = \{b_1, b_2, b_3\}$ respectively,

Let.

$F(e_1^{\#}b_1) = \{c_1, c_2, c_4, c_5\}$, $F(e_1^{\#}b_2) = \{c_2, c_3\}$, $F(e_1^{\#}b_3) = \{c_2, c_4\}$, $F(e_2^{\#}b_1) = \{c_2, c_3, c_4, c_5\}$, $F(e_2^{\#}b_2) = \{c_3\}$, $F(e_2^{\#}b_3) = \{c_2, c_4\}$, $F(e_3^{\#}b_1) = \{c_1, c_2, c_3, c_4, c_5\}$, $F(e_3^{\#}b_2) = \{c_1, c_3, c_5\}$, $F(e_3^{\#}b_3) = \{c_2, c_4\}$.

Let.

$G(e_1^{\#}b_1) = \{c_1, c_3, c_4, c_5\}$, $G(e_1^{\#}b_2) = \{c_3, c_5\}$, $G(e_1^{\#}b_3) = \{c_2, c_4\}$, $G(e_2^{\#}b_1) = \{c_3, c_4, c_5\}$

$G(e_2^{\#}b_2) = \{c_2, c_3, c_4\}$, $G(e_2^{\#}b_3) = \{c_2, c_4, c_5\}$, $G(e_3^{\#}b_1) = \{c_2, c_3, c_4\}$, $G(e_3^{\#}b_2) = \{c_1, c_2, c_4\}$

$G(e_3^{\#}b_3) = \{c_2, c_4\}$.

Then $(F, M^{\#}B) \wedge (G, N^{\#}B) = (H_{0,} M^{\#}B \wedge N^{\#}B)$

Where,

$H_0(e_1^{\#}b_1 \wedge e_1^{\#}b_2) = \{c_5\}$, $H_0(e_1^{\#}b_1 \wedge e_2^{\#}b_3) = \{c_2, c_4, c_5\}$, $H_0(e_1^{\#}b_1 \wedge e_2^{\#}b_2) = \{c_2, c_4\}$, $H_0(e_2^{\#}b_1 \wedge e_1^{\#}b_1) = \{c_3, c_4, c_5\}$, $H_0(e_2^{\#}b_1 \wedge e_3^{\#}b_1) = \{c_2, c_3, c_4\}$, $H_0(e_2^{\#}b_2 \wedge e_1^{\#}b_3) = \Phi$, $H_0(e_1^{\#}b_3 \wedge e_2^{\#}b_3) = \{c_2, c_4\}$, $H_0(e_3^{\#}b_2 \wedge e_1^{\#}b_3) = \Phi$, $H_0(e_3^{\#}b_1 \wedge e_3^{\#}b_3) = \{c_2, c_4\}$, $H_0(e_2^{\#}b_3 \wedge e_3^{\#}b_2) = \{c_2, c_4\}$.

Definition 3.5. OR" Operation on Two Deterministic Soft Set

If $(F, M^{\#}B)$ and $(G, N^{\#}B)$ are two deterministic soft sets then "$(F, M^{\#}B)$ OR $((G, N^{\#}B))$" denoted by $(F, M^{\#}B) \vee (G, NB)$ is defined by: $(F, M^{\#}B) \vee (G, N^{\#}B) = (P_0, M^{\#}B \vee N^{\#}B)$ where $P_0(\alpha, \beta) = F(\alpha) \cup G(\beta)$, $\forall (\alpha, \beta) \in M^{\#}B \vee N^{\#}B$.

Example 3.3. Consider **example 3.2.** above, we see that $(F, M^{\#}B) \vee (G, NB) = (P_0, M^{\#}B \vee N^{\#}B)$.

Where,

$P_0(e_1^{\#}b_1 \vee e_1^{\#}b_2) = U$, $P_0(e_1^{\#}b_1 \vee e_2^{\#}b_3) = \{c_1, c_2, c_4, c_5\}$, $P_0(e_1^{\#}b_1 \vee e_2^{\#}b_2) = U$, $P_0(e_2^{\#}b_1 \vee e_1^{\#}b_1) = U$, $P_0(e_2^{\#}b_1 \vee e_3^{\#}b_1) = U$, $P_0(e_2^{\#}b_2 \vee e_1^{\#}b_3) = \{c_2, c_3, c_4\}$, $P_0(e_1^{\#}b_3 \vee e_2^{\#}b_3) = \{c_2, c_4,\}$, $P_0(e_3^{\#}b_2 \vee e_1^{\#}b_3) = U$, $P_0(e_3^{\#}b_1 \vee e_3^{\#}b_3) = U$, $P_0(e_2^{\#}b_3 \vee e_3^{\#}b_2) = \{c_1, c_2, c_4,\}$.

Proposition 3.1.

i. $((F, M^{\#}B) \vee (G, N^{\#}B))^c = (F, M^{\#}B)^c \wedge (G, N^{\#}B)^c$.
ii. $((F, M^{\#}B) \wedge (G, N^{\#}B))^c = (F, M^{\#}B)^c \vee (G, N^{\#}B)^c$.

Proof.

i. Suppose that $(F, M^{\#}B) \vee (G, N^{\#}B) = (P_0, M^{\#}B \times N^{\#}B)$.

Therefore,

$$((F, M^{\#}B) \vee (G, N^{\#}B))^{c} = (P_0, M^{\#}B \times N^{\#}B)^{c} = (P_0{}^{c}, \urcorner(M^{\#}B \times N^{\#}B)).$$

Now,

$$(F, M^{\#}B)^{c} \wedge (G, N^{\#}B)^{c} = (F^{c}, \urcorner M^{\#}B) \wedge (G^{c}, \urcorner N^{\#}B),$$

$$= (Q_0, \urcorner M^{\#}B \times \urcorner N^{\#}B) \quad \text{where, } Q_0, (x,y) = F^{c}(x) \cap G^{c}(y),$$

$$= (Q_0, \urcorner(M^{\#}B \times N^{\#}B)).$$

Now, take $(\urcorner\alpha, \urcorner\beta) \in \urcorner(M^{\#}B \times N^{\#}B)$

Therefore,

$$\begin{aligned} P_0{}^{c}(\urcorner\alpha, \urcorner\beta) &= U - P(\alpha, \beta), \\ &= U - [F(\alpha) \cup G(\beta)] \\ &= [U - F(\alpha)] \cap [U - G(\beta)] \\ &= F^{c}(\urcorner\alpha) \cap G^{c}(\urcorner\beta), \\ &= Q_0, (\urcorner\alpha, \urcorner\beta) \end{aligned}$$

Remark 3.1. This implies that $P_0{}^{c}$ and Q_0, are same. Hence, proved.

ii. Suppose that $(F, M^{\#}B) \wedge (G, N^{\#}B) = (H_0, M^{\#}B \times N^{\#}B)$

Therefore,

$$((F, M^{\#}B) \wedge (G, N^{\#}B))^{c} = (H_0, M^{\#}B \times N^{\#}B)^{c} = (H_0{}^{c}, \urcorner M^{\#}B \times N^{\#}B)).$$

Now,

$$(F, M^{\#}B)^{c} \vee (G, N^{\#}B)^{c} = (F^{c}, \urcorner M^{\#}B) \vee (G^{c}, \urcorner M^{\#}B)$$

$$= (I_0, \urcorner M^{\#}B \times \urcorner N^{\#}B)$$

Where,

$$I_0(x,y) = F^{c}(x) \cup G^{c}(y),$$

$$= (I_0, \urcorner(M^{\#}B \times N^{\#}B))$$

Now, take $(\urcorner\alpha, \urcorner\beta) \in \urcorner(M^{\#}B \times N^{\#}B)$

Therefore,

$$\begin{aligned} H_0{}^{c}(\urcorner\alpha, \urcorner\beta) &= U - H_0(\urcorner\alpha, \urcorner\beta), \\ &= U - [F(\alpha) \cap G(\beta) \\ &= [U - F(\alpha)] \cup [U - G(\beta)] \\ &= F^{c}(\urcorner\alpha) \cup G^{c}(\urcorner\beta) \\ &= I_0(\urcorner\alpha, \urcorner\beta) \end{aligned}$$

Remark 3.2. This shows that $H_0{}^{c}$ and I_0 are same.

Definition 3.6. Union of two deterministic soft sets $(F, M^{\#}B)$ and $(G, N^{\#}B)$ over the common universe U is the deterministic soft set (P_0, O^{*}), where $O^{*} = M^{\#}B \cup N^{\#}B$ and $\forall\, e^{*} \in O^{*}$.

$$\begin{aligned} P_0(e^{*}) &= F(e^{*}) && \text{if } e^{*} \in M^{\#}B - N^{\#}B \\ P_0(e^{*}) &= G(e^{*}) && \text{if } e^{*} \in N^{\#}B - M^{\#}B \\ &= F(e^{*}) \cup G(e^{*}), && \text{if } e^{*} \in M^{\#}B \cap N^{\#}B \end{aligned}$$

We write $(F, M^{\#}B) \,\tilde{\cup}\, (G, N^{\#}B) = (P_0, O^{*})$

Consider **example 3.2.** above,

$(F, M^{\#}B)$ $\tilde{\cup}$ $(G, N^{\#}B)$ $= (P_0, O_{ij}^*) = (P_0, e_{ij}^*)$, where $i = 1, 2, 3, \ldots m$ and $j = 1, 2, 3, \ldots n$ such that $(F, M^{\#}B)$ $\in (F, D^*)$.

Where.

$P_0(e_{11\times12}^*) = U$, $P_0(e_{11\times23}^*) = \{c_1, c_2, c_4, c_5\}$, $P_0(e_{11\times22}^*) = U$, $P_0(e_{21\times11}^*) = U$, $P_0(e_{21\times31}^*) = U$, $P_0(e_{22\times13}^*) = \{c_2, c_3, c_4\}$, $P_0(e_{13\times23}^*) = \{c_2, c_4\}$, $P_0(e_{32\times13}^*) = U$, $P_0(e_{31\times33}^*) = U$, $P_0(e_{23\times32}^*) = \{c_1, c_2, c_4\}$.

Definition 3.7. Intersection of two deterministic soft sets $(F, M^{\#}B)$ and $(G, N^{\#}B)$ over the common universe U is the deterministic soft set (H_0, O^*), where $O^* = M^{\#}B \cap N^{\#}B$ and $\forall\, e^* \in O^*$.

$$\begin{aligned} P_0(e^*) &= F(e^*) && \text{if } e^* \in M^{\#}B - N^{\#}B \\ P_0(e^*) &= G(e^*) && \text{if } e^* \in N^{\#}B - M^{\#}B \\ &= F(e^*) \cap G(e^*), && \text{if } e^* \in M^{\#}B \cap N^{\#}B \end{aligned}$$

We write $(F, M^{\#}B) \tilde{\cap} (G, N^{\#}B) = (H_0, O^*)$

Consider **example 3.2.** above,

$(F, M^{\#}B) \tilde{\cap} (G, N^{\#}B) = (H_0, O_{ij}^*) = (H_0, e_{ij}^*)$, where $i = 1, 2, 3, \ldots m$ and $j = 1, 2, 3, \ldots n$

Where.

$H_0(e_{11\times12}^*) = \{c_5\}$, $H_0(e_{11\times23}^*) = \{c_2, c_4, c_5\}$, $H_0(e_{11\times22}^*) = \{c_2, c_4\}$, $H_0(e_{21\times11}^*) = \{c_3, c_4, c_5\}$, $H_0(e_{21\times31}^*) = \{c_2, c_3, c_4\}$, $H_0(e_{22\times13}^*) = \Phi$, $H_0(e_{13\times23}^*) = \{c_2, c_4\}$, $H_0(e_{32\times13}^*) = \Phi$, $H_0(e_{31\times33}^*) = \{c_2, c_4\}$, $H_0(e_{23\times32}^*) = \{c_2, c_4\}$.

Proposition 3.2.

i. $((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B))^c = (F, M^{\#}B)^c \tilde{\cup} (G, N^{\#}B)^c$

ii. $((F, M^{\#}B) \tilde{\cap} (G, N^{\#}B))^c = (F, M^{\#}B)^c \tilde{\cap} (G, N^{\#}B)^c$

Proof.

i. Suppose that $(F, M^{\#}B) \tilde{\cup} (G, N^{\#}B) = (P_0, M^{\#}B \cup N^{\#}B)$, where

$$\begin{aligned} P_0(\gamma) &= F(\gamma), && \text{if } \gamma \in M^{\#}B - N^{\#}B, \\ &= G(\gamma), && \text{if } \gamma \in N^{\#}B - M^{\#}B \\ &= F(\gamma) \cup G(\gamma), && \text{if } \gamma \in M^{\#}B \cap N^{\#}B \end{aligned}$$

Therefore,

$$\begin{aligned} ((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B))^c &= (P_0, M^{\#}B \cup N^{\#}B)^c \\ &= (P_0{}^c, \rceil M^{\#}B \cup \rceil N^{\#}B) \end{aligned}$$

Now, $P_0{}^c(\rceil\gamma) = U - P_0(\gamma)$, $\forall\, \rceil\gamma \in \rceil M^{\#}B \cup \rceil N^{\#}B$

Therefore,

$$P_0{}^c(\rceil\gamma) = F^c(\rceil\gamma), \qquad \text{if } \rceil\gamma \in \rceil M^{\#}B - \rceil N^{\#}B,$$

$= G^c(\rceil\gamma)$ if $\rceil\gamma \in \rceil N^\# B - \rceil M^\# B$,

$= F^c(\rceil\gamma) \cup G^c(\rceil\gamma)$ if $\rceil\gamma \in \rceil M^\# B \cap \rceil N^\# B$,

Again,

$(F, M^\# B)^c \tilde{\cup} (G, N^\# B)^c = (F, M^\# B) \tilde{\cup} (G, N^\# B)$

$= (Q_0, \rceil M^\# B \cup \rceil N^\# B)$ say

Where,

$Q_0(\rceil\gamma) = F^c(\rceil\gamma)$ if $\rceil\gamma \in \rceil M^\# B$ - $\rceil N^\# B$,

$= G^c(\rceil\gamma)$ if $\rceil\gamma \in \rceil N^\# B$ - $\rceil M^\# B$,

$= F^c(\rceil\gamma) \cup G^c(\rceil\gamma)$ if $\rceil\gamma \in \rceil N^\# B \cap \rceil M^\# B$,

Remark 3.3. This shows that ${P_0}^c$ and Q_0 are same.

ii. Suppose that $((F, M^\# B) \tilde{\cap} (G, N^\# B)) = (H_0, M^\# B \cap N^\# B)$

Therefore,

$((F, M^\# B) \tilde{\cap} (G, N^\# B))^c = (H_0, \rceil M^\# B \cap \rceil N^\# B)$

Now,

$((F, M^\# B)^c \tilde{\cap} (G, N^\# B)^c = (F^c, \rceil\gamma) \tilde{\cap} (G^c, \rceil\gamma)$

$= (I_0, \rceil M^\# B \cap \rceil N^\# B)$

Where,

$\forall \rceil\gamma \in (\rceil M^\# B \cap \rceil N^\# B)$, we have

${I_0}^c(\rceil\gamma) = F^c(\rceil\gamma)$ or $G^c(\rceil\gamma)$,

$= F(\rceil\gamma)\ or\ G(\rceil\gamma)\ where\ \forall\ \gamma \in M^\# B \cap N^\# B$

$= H_0(\gamma)$

$= {H_0}^c(\rceil\gamma)$

Remark 3.4. This shows that I_0 and ${H_0}^c$ are same function, hence proved.

Proposition 3.3. If $(F, M^\# B)$, $(G, N^\# B)$, and $(J, R^\# B)$ are three deterministic soft sets over U, then.

i. $(F, M^\# B) \tilde{\cup} ((G, N^\# B) \tilde{\cup} (J, R^\# B)) = ((F, M^\# B) \tilde{\cup} (G, N^\# B)) \tilde{\cup} (J, R^\# B)$,

ii. $(F, M^\# B) \tilde{\cap} ((G, N^\# B) \tilde{\cap} (J, R^\# B)) = ((F, M^\# B) \tilde{\cap} (G, N^\# B)) \tilde{\cap} (J, R^\# B)$,

iii. $(F, M^{\#}B) \tilde{\cup} ((G, N^{\#}B) \tilde{\cap} (J, R^{\#}B)) = ((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B)) \tilde{\cap} ((F, M^{\#}B) \tilde{\cup} (J, R^{\#}B))$,

iv. $(F, M^{\#}B) \tilde{\cap} ((G, N^{\#}B) \tilde{\cup} (J, R^{\#}B)) = ((F, M^{\#}B) \tilde{\cap} (G, N^{\#}B)) \tilde{\cup} ((F, M^{\#}B) \tilde{\cap} (J, R^{\#}B))$.

From proposition 3.3

Iii. $(F, M^{\#}B) \tilde{\cup} ((G, N^{\#}B) \tilde{\cap} (J, R^{\#}B)) = ((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B)) \tilde{\cap} ((F, M^{\#}B) \tilde{\cup} (J, R^{\#}B))$,

Proof.

Let α be an arbitrary subset of:
$(F, M^{\#}B) \tilde{\cup} ((G, N^{\#}B) \tilde{\cap} (J, R^{\#}B))$

Then,

$\alpha \in (F, M^{\#}B) \tilde{\cup} ((G, N^{\#}B) \tilde{\cap} (J, R^{\#}B)) = \alpha \in (F, M^{\#}B)$ or $\alpha \in (G, N^{\#}B) \tilde{\cap} (J, R^{\#}B)$

$= \alpha \in (F, M^{\#}B)$ or $\alpha \in (G, N^{\#}B)$ and $\alpha \in (J, R^{\#}B)$

$= (\alpha \in (F, M^{\#}B)$ or $\alpha \in (G, N^{\#}B))$ and $\alpha \in (J, R^{\#}B)$

Therefore "OR" distributes "AND"

$= \alpha \in (F, M^{\#}B) \tilde{\cup} (G, N^{\#}B)$ and $\alpha \in (F, M^{\#}B) \tilde{\cup} (J, R^{\#}B)$

$= \alpha \in ((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B)) \tilde{\cap} \alpha \in ((F, M^{\#}B) \tilde{\cup} (J, R^{\#}B))$

Therefore,

$(F, M^{\#}B) \tilde{\cup} ((G, N^{\#}B) \tilde{\cap} (J, R^{\#}B)) \subseteq (F, M^{\#}B) \tilde{\cup} (G, N^{\#}B) \tilde{\cap} (F, M^{\#}B) \tilde{\cup} (J, R^{\#}B)$ ....(i)

Again let β be an arbitrary subset of:

Then,

$\beta \in ((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B)) \tilde{\cap} ((F, M^{\#}B) \tilde{\cup} (J, R^{\#}B))$

$= \beta \in ((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B))$ and $\beta \in ((F, M^{\#}B) \tilde{\cup} (J, R^{\#}B))$

$= (\beta \in ((F, M^{\#}B)$ or $(\beta \in (G, N^{\#}B))$ and $(\beta \in (F, M^{\#}B))$ or $\in (J, R^{\#}B))$

$= \beta \in ((F, M^{\#}B)$ or $(\beta \in (G, N^{\#}B))$ and $\beta \in (J, R^{\#}B)$

Therefore, "OR" distributes "AND"

$= \beta \in (F, M^{\#}B)$ or $\beta \in ((G, N^{\#}B) \tilde{\cap} (J, R^{\#}B))$

$= \beta \in ((F, M^{\#}B) \tilde{\cup} \beta \in ((G, N^{\#}B) \tilde{\cap} (J, R^{\#}B))$

Therefore,

$((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B)) \tilde{\cap} ((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B)) \subseteq (F, M^{\#}B) \tilde{\cup} ((G, N^{\#}B) \tilde{\cap} (J, R^{\#}B))$... (ii).

From equation (i) and (ii) we get.

$(F, M^{\#}B) \tilde{\cup} ((G, N^{\#}B) \tilde{\cap} (J, R^{\#}B)) = ((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B)) \tilde{\cap} ((F, M^{\#}B) \tilde{\cup} (J, R^{\#}B))$,

` Hence proved.

From proposition 3.3

Iv. $(F, M^{\#}B) \tilde{\cap} ((G, N^{\#}B) \tilde{\cup} (J, R^{\#}B)) = ((F, M^{\#}B) \tilde{\cap} (G, N^{\#}B)) \tilde{\cup} ((F, M^{\#}B) \tilde{\cap} (J, R^{\#}B))$

Proof.

i. Let α be an arbitrary subset of:

$(F, M^{\#}B) \tilde{\cap} ((G, N^{\#}B) \tilde{\cup} (J, R^{\#}B))$

Then,

$\alpha \in (F, M^{\#}B) \tilde{\cap} ((G, N^{\#}B) \tilde{\cup} (J, R^{\#}B)) = \alpha \in (F, M^{\#}B)$ and $\alpha \in (G, N^{\#}B) \tilde{\cup} (J, R^{\#}B)$

$= \alpha \in (F, M^{\#}B)$ and $\alpha \in (G, N^{\#}B)$ or $\alpha \in (J, R^{\#}B)$

$= \alpha \in (F, M^{\#}B)$ and $\alpha \in (G, N^{\#}B)$ or $\alpha \in (F, M^{\#}B)$ and $\alpha \in (J, R^{\#}B)$

Therefore "OR" distributes "AND"

$= \alpha \in (F, M^{\#}B) \tilde{\cap} (G, N^{\#}B)$ or $\alpha \in (F, M^{\#}B) \tilde{\cap} (J, R^{\#}B)$

$= \alpha \in ((F, M^{\#}B) \tilde{\cup} (G, N^{\#}B)) \tilde{\cap} \alpha \in ((F, M^{\#}B) \tilde{\cup} (J, R^{\#}B))$

Therefore,

$(F, M^{\#}B) \tilde{\cap} ((G, N^{\#}B) \tilde{\cup} (J, R^{\#}B)) \subseteq (F, M^{\#}B) \tilde{\cup} (G, N^{\#}B) \tilde{\cap} (F, M^{\#}B) \tilde{\cup} (J, R^{\#}B)$ ....(iii)

Again let β be an arbitrarysubset of:

Then,

$\beta \in ((F, M^{\#}B) \tilde{\cap} (G, N^{\#}B)) \tilde{\cup} ((F, M^{\#}B) \tilde{\cap} (J, R^{\#}B))$

$= \beta \in ((F, M^{\#}B) \tilde{\cap} (G, N^{\#}B))$ or $\beta \in ((F, M^{\#}B) \tilde{\cap} (J, R^{\#}B))$

$= (\beta \in ((F, M^{\#}B)$ and $(\beta \in (G, N^{\#}B))$ or $((\beta \in (F, M^{\#}B))$ and $\in (J, R^{\#}B))$

$= \beta \in ((F, M^{\#}B)$ and $(\beta \in (G, N^{\#}B)$ or $\beta \in (J, R^{\#}B)$

Therefore, "OR" distributes "AND"

$= \beta \in (F, M^{\#}B)$ and $\beta \in ((G, N^{\#}B) \tilde{\cup} (J, R^{\#}B))$

$= \beta \in ((F, M^{\#}B) \tilde{\cap} \beta \in ((F, M^{\#}B) \tilde{\cup} (J, R^{\#}B))$

Therefore,

$((F, M^{\#}B) \tilde{\cap} (G, N^{\#}B)) \tilde{\cup} ((F, M^{\#}B) \tilde{\cap} (G, N^{\#}B)) \subseteq (F, M^{\#}B) \tilde{\cup} ((G, N^{\#}B) \tilde{\cap} (J, R^{\#}B))$... (iv).

From equation (iii) and (iv) we get.

$(F, M^{\#}B) \tilde{\cap} ((G, N^{\#}B) \tilde{\cup} (J, R^{\#}B)) = ((F, M^{\#}B) \tilde{\cap} (G, N^{\#}B)) \tilde{\cup} ((F, M^{\#}B) \tilde{\cap} (J, R^{\#}B))$,

Hence proved.

3. Application of Deterministic Soft Set

Maji et al. (2002) applied the theory of soft sets to solve a decision making problem. In this section, we present an application of deterministic soft set theory in a decision making problem. The problem we consider is as below.

Example 4.1.

Assume an individual is to buy the following commodities according to his order of priorities.

There are seven (7) commodities which form the universe.

$U = \{c_{1,} c_{2,} c_3, c_4, c_{5,} c_{6,} c_7\}$, where, $c_1 = Gold$, $c_2 = House$, $c_3 = Car$, $c_4 = Grains$, $c_5 = Diamond$, $c_6 = Clothing$, $c_7 = Medicine$. The individual considers a set of parameters: $E = \{e_{1,} e_{2,} e_3, e_4\}$. Where, $e_1 = Expensive$, $e_2 = Pleasant$, $e_3 = Valuable$, e_4 = Consumable, and there is the determinant: $B = \{b_{1,} b_{2,} b_3\}$. Where, $b_1 = Essential$, $b_2 = Neccesity$, b_3 = Desire

Note: That is "B" is the set of deterministic parameters.

Let $e_i^{\#} b_j \in d_{ij}^* \; \forall\, e, b \subseteq d.$ where $i = 1, 2, 3, \ldots m$ and $j = 1, 2, 3, \ldots n$ such that $(F, M^{\#}B) \in (F, D^*)$.

$(\boldsymbol{F}, \boldsymbol{D}^*)$ = $(d_{11}^+, 1) = \{c_{1,} c_{4,} c_{6,} c_7\}$, $(d_{12}^+, 1) = \{c_{1,} c_{2,} c_3, c_{4,} c_{6,} c_7\}$, $(d_{13}^+, 1) = \{c_{1,} c_{2,} c_3, c_{4,} c_{5,} c_6\}$, $(d_{21}^+, 1) = \{c_{2,} c_{4,} c_{6,} c_7\}$, $(d_{22}^+, 1) = \{c_{1,} c_{2,} c_3, c_{4,} c_{6,} c_7\}$, $(d_{23}^+, 1) = \{c_{1,} c_{2,} c_3, c_{4,} c_5\}$, $(d_{31}^+, 1) = \{c_{2,} c_{3,} c_4, c_{6,} c_7\}$, $(d_{32}^+, 1) = U$, $(d_{33}^+, 1) = \{c_{1,} c_{2,} c_3, c_{5,} c_6\}$, $(d_{41}^+), 1) = \{c_{4,} c_6, c_7\}$, $(d_{42}^+, 1) = \{c_{4,} c_6, c_7\}$, $(d_{43}^+, 1) = \{c_4\}$, $(d_{11}^-, 0) = \{c_1, c_3, c_5\}$, $(d_{12}^-, 0) = \{c_5\}$, $(d_{13}^-, 0) = \{c_7\}$, $(d_{21}^-, 0) = \{c_3, c_5\}$

$(d_{22}^-, 0) = \{c_5\}$, $(d_{23}^-, 0) = \{c_6, c_7\}$, $(d_{31}^-, 0) = \{c_{1,} c_5\}$, $(d_{32}^-, 0) = \Phi$, $(d_{33}^-, 0) = \{c_4, c_7\}$, $(d_{41}^-, 0) = \{c_{1,} c_{2,} c_3, c_5\}$, $(d_{42}^-, 0) = \{c_{1,} c_{2,} c_3, c_5\}$, $(d_{43}^-, 0) = \{c_{1,} c_{2,} c_3, c_{5,} c_{6,} c_7\}$

In table 1 and 2, we present the agree deterministic soft set and disagree deterministic soft set respectively, such that $C_I \in F_1(\varepsilon)$ then $C_{ij} = 1$ otherwise $C_{ij} = 0$, and if $C_I \in F_0(\varepsilon)$ then $C_{ij} = 1$ otherwise $C_{ij} = 0$ where C_{ij} are entries in table 1 and 2.

The following algorithm may be used by the consumer in buying the commodities base on his order of priority.

Algorithm

i. Input the deterministic soft set (F, D^*).
ii. Find an agree deterministic soft set and a disagree deterministic soft set.
iii. Find $L_x = \sum C_{ij}$ for agree deterministic soft set.
iv. Find $K_y = \sum C_{ij}$ for disagree deterministic soft set.
v. Find $\boldsymbol{T_z} = \frac{\boldsymbol{L_{x} - K_y}}{\boldsymbol{N_c}} \times \mathbf{100}\%$ Where, T_z= Total outcome, L_x= Agree outcome, K_y= Disagrre outcome, N_c= Number of chances.
vi. Arrange the commodities in order of priorities.

Table 2: Agree Deterministic Soft Set

U	c_1	c_2	c_3	c_4	c_5	c_6	c_7
(d_{11}^{+})	0	1	0	1	0	1	1
(d_{12}^{+})	1	1	1	1	0	1	1
(d_{13}^{+})	1	1	1	1	1	1	0
(d_{21}^{+})	0	1	0	1	0	1	1
$((d_{22}^{+})$	1	1	1	1	0	1	1
(d_{23}^{+})	1	1	1	1	1	0	0
(d_{31}^{+})	0	1	1	1	0	1	1
(d_{32}^{+})	1	1	1	1	1	1	1
(d_{33}^{+})	1	1	1	0	1	1	0
(d_{41}^{+})	0	0	0	1	0	1	1
(d_{42}^{+})	0	0	0	1	0	1	1
(d_{43}^{+})	0	0	0	1	0	0	0
$L_x = \sum C_{ij}$	6	9	7	11	4	10	8

Table 3: Disagree Deterministic Soft Set

U	c_1	c_2	c_3	c_4	c_5	c_6	c_7
(d_{11}^{-})	1	0	1	0	1	0	0
(d_{12}^{-})	0	0	0	0	1	0	0
(d_{13}^{-})	0	0	0	0	0	0	1
(d_{21}^{-})	1	0	1	0	1	0	0
(d_{22}^{-})	0	0	0	0	1	0	0
(d_{23}^{-})	0	0	0	0	0	1	1
(d_{31}^{-})	1	0	0	0	1	0	0
(d_{32}^{-})	0	0	0	0	0	0	0
(d_{33}^{-})	0	0	0	1	0	0	1
(d_{41}^{-})	1	1	1	0	1	0	0
(d_{42}^{-})	1	1	1	0	1	0	0
(d_{43}^{-})	1	1	1	0	1	1	1
$K_y = \mathcal{E} C_{ij}$	6	3	5	1	8	2	4

Table 4: Total Outcomes

$L_x = \sum C_{ij}$	$K_y = \sum C_{ij}$	$T_z = \frac{L_x - K_y}{N_c} \times 100\%$
6	6	$T_1 = \frac{6-6}{12} \times 100 = 0\%$
9	3	$T_2 = \frac{9-3}{12} \times 100 = 50\%$
7	5	$T_3 = \frac{7-5}{12} \times 100 = 16.7\%$
11	1	$T_4 = \frac{11-1}{12} \times 100 = 83.3\%$
4	8	$T_5 = \frac{4-8}{12} \times 100 = -33.3\%$
10	2	$T_6 = \frac{10-2}{12} \times 100 = 66.7\%$
8	4	$T_7 = \frac{8-4}{12} \times 100 = 33.3\%$

Table 5: Scale of Preference

Arrange the Commodities in Order of Priorities.

COMMODITIES	%
c_4 = Grain	83.3%
c_6 = Clothing	66.7%
c_2 = House	50.0%
c_7 = Medicine	33.3%
c_3 = Car	16.3%
c_1 = Gold	0%
c_5 = Diamond	−33.3%

We realize that the individual uses the above algorithm in finding the commodity with optimal importance base on his order of priorities. Grain which is taking the highest percentage (83.3%) was first on his list, then he also realizes the need to buy clothes which is the second with 66.7%, there was also need for him to get an accommodation which occupies the third on the list with 50%, Medicine (33.3%) which is the fourth, Car (16.3%) which is the fifth, Gold (0%) which is the sixth and lastly diamond (-33.3%) which is the seventh.

4. Conclusion

As a new tool dealing with uncertainty, we have introduced the deterministic soft set theory which is more efficient and useful and also studied some of its properties. We also defined basic operations on deterministic soft sets, such as complement, union, intersection, OR and AND. The theory has been applied to solve problems on decision making. In future work, researchers can generalize this concept to interval-valued deterministic soft set and they also can develop it to deterministic soft expert set to give it more efficiency.

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