SCIENCE FORUM (JOURNAL OF PURE AND APPLIED SCIENCES) 25, (3) (2025), 258 - 280

Science Forum (Journal of Pure and Applied Sciences)

Journal Homepage: https://atbuscienceforum.com.ng/index.php/jpas

Mathematical Modeling of Banditry and Sustainable Pathways to Peace: Optimal Control through SDG-Based Intervention Strategies

Mammam Mamuda, Abdullahi Mohammed Baba, Chado Umar Doko

Department of Mathematics, School of Sciences. Niger State College of Education, Minna

Abstract

Banditry has emerged as one of Nigeria's most urgent security and developmental crises, destabilizing communities, disrupting education, undermining livelihoods, and eroding public trust in governance. This study develops a student-focused mathematical model that explicitly captures the dynamics of banditry while assessing optimal intervention strategies grounded in the Sustainable Development Goals (SDGs). In contrast to earlier models that address banditry primarily from broad socio-political perspectives, this research foregrounds the vulnerabilities of students and educational institutions—demonstrating how persistent insecurity threatens human capital development and national progress. The model presented in this study introduces three strategic control levers: security and justice enforcement (u_1), education and job creation (u_2), and rehabilitation and reintegration programs (u_3). Employing an epidemic-inspired compartmental framework, Pontryagin's Maximum Principle, and Forward-Backward Sweep numerical simulations, the study evaluates the "banditry reproduction number" across different intervention scenarios. Findings reveal that an integrated triple-strategy approach ($u_1+u_2+u_3$) is most effective, rapidly suppressing banditry while fostering long-term reintegration of offenders. In contexts constrained by limited resources, the dual strategy combining security enforcement with education and employment opportunities (u_1+u_2) emerges as the most practical balance, disrupting recruitment pipelines while simultaneously addressing root causes. Numerical results highlight key dynamics: enforcement measures reduce the pool of active bandits, educational and employment programs block the inflow of new recruits, and rehabilitation initiatives channel former offenders into productive and peaceful reintegration. The analysis demonstrates that no single measure is sufficient; only coordinated, multi-dimensional strategies can dismantle criminal structures, rebuild community resilience, and restore stability. By explicitly linking outcomes to SDG 4 (Quality Education), SDG 8 (Decent Work and Economic Growth), and SDG 16 (Peace, Justice, and Strong Institutions), this research provides a policy-relevant, evidence-based framework for designing sustainable intervention strategies. Beyond Nigeria, the model offers broader insights into addressing violent criminality in fragile contexts across West Africa, positioning mathematical modeling as a valuable tool for guiding peacebuilding and development policy.

Keywords

Banditry, Mathematical Modeling, Optimal Control, SDGs, Education, Security, Reintegration, Nigeria.

Corresponding Author: Mammam Mamuda, Department of Mathematics, School of Sciences. Niger State College of Education, Minna, Email kutigibaba@gmail.com

Introduction

Banditry in West Africa is a deeply entrenched challenge, shaped by centuries of historical, political, and socio-economic forces. Its roots can be traced to the legacies of the trans-Saharan slave trade, colonial disruptions, and the persistent weaknesses of post-colonial governance. These factors created fragile state institutions and exacerbated socio-economic inequalities, leaving rural communities vulnerable to cycles of violence and exploitation. Across the Sahel, banditry has been fueled by poverty, resource scarcity, and governance failures. In Niger Republic, persistent insecurity in Tillaberi and Tahoua is linked to widespread poverty and weak state presence (Abdullahi, 2019). Northern Mali faces parallel challenges, where fragile institutions and recurrent insurgencies push marginalized pastoralists into extremist and criminal networks. In Mauritania, disputes over scarce resources such as water and grazing land have escalated into armed criminality (Accord, 2022).

Nigeria, however, represents the epicenter of rural banditry in the sub-region. A combination of widespread poverty, porous borders, arms proliferation, and governance failures has created fertile ground for armed groups. Unemployed youths from farming and herding backgrounds are frequently recruited into gangs that thrive on fear, extortion, and instability (Adeoye, 2018; Bagu & Smith, 2017; Okoli & Ugwu, 2019; Rosenje & Adeniyi, 2021). Zamfara State is the most severe hotspot, hosting an estimated 10,000 armed bandits. Their operations have led to over 12,000 deaths, 3,600 kidnappings, the theft of 250,000 livestock, and the displacement of hundreds of thousands of people (Rufa'i, 2021). Illegal mining activities and the complicity of corrupt actors—including local elites and some security officials—have further entrenched the crisis (Mustapha, 2019; Ogbonnaya, 2020; Global Security, 2022).

Banditry manifests in multiple forms: armed robbery, cattle rustling, rural raids, and mass abductions. A defining moment came with the abduction of 276 Chibok schoolgirls in 2010, which not only attracted global attention but also marked the expansion of abduction-for-ransom as a central strategy. Subsequent mass kidnappings have terrorized communities, forced the shutdown of schools, and disrupted local economies (Neack, 2017; Usman, 2021). Beyond immediate economic losses, persistent insecurity erodes development gains, displaces farmers, weakens infrastructure, and undermines state-led interventions (Wallace & Silander, 2018; Mohammed et al., 2021). Despite multiple countermeasures—including military offensives, peace dialogues, and community policing—government responses have produced limited results.

Recent scholarship increasingly applies mathematical modeling to analyze and design responses to banditry. Such models underscore the importance of combining security interventions with socio-economic measures like rehabilitation, employment, and deradicalization. For example, Aniyam et al. (2018) and Lawal et al. (2023) demonstrate how access to jobs and reintegration programs reduces the appeal of criminal networks, while Tasiu et al. (2024) emphasize the necessity of coupling state-led security operations with socio-economic reforms. Mathematical tools such as Pontryagin's Maximum Principle and Forward-Backward Sweep simulations allow policymakers to evaluate intervention strategies in a quantitative, evidence-driven manner before implementation (Momo et al., 2023).

Nigeria's banditry crisis mirrors global terrorism and organized crime patterns. Groups such as Boko Haram and Fulani extremists exploit vulnerabilities similar to those harnessed by Al-Qaeda, ISIS, and Al-Shabaab. In 2017, Boko Haram was responsible for more than 1,000 deaths, while Fulani

extremists caused over 300 deaths (Global Terrorism Index, 2018). Such figures illustrate how organized armed violence disrupts governance and development while blurring the boundaries between terrorism, insurgency, and organized crime.

The crisis has also severely undermined Nigeria's tertiary education sector. Surveys indicate that more than 65% of students struggle with concentration, while nearly 70% report heightened anxiety after nearby attacks (Damilola, 2025). A probit regression study confirms a strong negative relationship between insecurity and institutional development in North-Central universities (Okoye & Nwaka-Nwandu, 2023). The consequences are stark: declining enrollment, diversion of funds from infrastructure to security, frequent campus closures, and mass student withdrawals (Idowu & Eze, 2024). More than 11,500 schools have been closed in high-risk regions, generating extensive learning losses and trauma among students (UNICEF, 2022; Shockden, 2023).

Emerging epidemic-inspired models provide new insights by conceptualizing recruitment into armed groups as infection and rehabilitation as recovery. These frameworks employ threshold analysis and optimal control methods to design multi-pronged interventions (Abdulrahman et al., 2024; Yakubu & Abah, 2023). However, most existing models overlook the unique vulnerabilities of tertiary education institutions—including hostel insecurity, psychosocial trauma, and the long-term erosion of human capital. Moreover, they rarely integrate Sustainable Development Goal (SDG)-aligned interventions such as quality education (SDG 4), youth employment (SDG 8), and peace, justice, and strong institutions (SDG 16).

This study addresses these gaps by proposing a student-centered mathematical model of banditry. The model estimates the "banditry reproduction number" under different intervention strategies and evaluates the optimal mix of controls necessary to restore safe and productive learning environments. By aligning with specific SDG targets, the study seeks to provide an evidence-based framework for peacebuilding and sustainable development in Nigeria and the wider West African sub-region.

Model Formulation

This study develops a student-centered epidemic-inspired mathematical model designed to capture the dynamics of how students and young people are recruited into banditry, how they exit through justice or rehabilitation pathways, and how institutional support structures can mitigate these risks. The model integrates three SDG-aligned intervention strategies that serve as control levers:

1. Security and Justice (u_1): Military enforcement, policing, legal prosecution, and community security arrangements to suppress active banditry.
2. Education and Jobs (u_2): Strengthening access to education, skills development, and youth employment opportunities to reduce susceptibility to recruitment.
3. Rehabilitation and Reintegration (u_3): Programs targeting ex-bandits with deradicalization, psychosocial therapy, and reintegration into peaceful livelihoods.

The structure of the model is informed by epidemic modeling principles, where recruitment into banditry parallels infection, persistence corresponds to active carriers, and rehabilitation represents recovery. Transition probabilities are shaped by both

socio-economic vulnerabilities and institutional interventions.

Model Purpose

The purpose of the model is to provide a quantitative and policy-relevant framework for understanding the emergence, persistence, and decline of banditry. Specifically, the model:

- Tests the impact of the three SDG-based levers—security and justice (u_1), education and jobs (u_2), and rehabilitation and reintegration (u_3)—on the spread and suppression of banditry.
- Evaluates the "banditry reproduction number" as a threshold indicator to determine whether interventions can prevent or eliminate recruitment pipelines.
- Identifies optimal policy mixes that balance immediate suppression with long-term prevention, particularly in resource-limited settings.
- Provides insights for education-specific interventions, highlighting how protecting schools and fostering resilience among students can reduce vulnerability to recruitment.

By combining mathematical rigor with development policy frameworks, this study advances a novel pathway to peacebuilding. It demonstrates that no single measure is sufficient, but integrated, SDG-aligned strategies can close recruitment channels, dismantle criminal structures, and rebuild resilient communities.

Table 1. List of State Variables

State Variable	Description
S(t)	Susceptible individuals who are at risk of being influenced or recruited into banditry.
E(t)	Exposed individuals who are vulnerable and under pressure but not yet fully recruited.
B(t)	Active bandits engaged in violent or criminal activity.
I(t)	Informants or collaborators who support bandits and facilitate recruitment.
R(t)	Rehabilitated or reintegrated individuals who have exited banditry through interventions.

Table 2. List of Control Variables and Their Roles

Control	Description	Mechanism of Action	Related SDG
u1	**Security and Justice**	Enhances policing, surveillance, intelligence, and legal enforcement; disrupts bandit–informant loops; increases exit of active bandits.	SDG 16 (Peace, Justice, and Strong Institutions)
u2(t	**Education and Jobs**	Expands schooling, mentorship, and livelihood opportunities; reduces vulnerability of exposed individuals; reintegrates informants back into society.	SDG 4 (Quality Education), SDG 8 (Decent Work and Economic Growth)
u3(t)	**Rehabilitation and Reintegration**	Provides amnesty, counseling, vocational training, and social reintegration programs; accelerates the transition of bandits to rehabilitated states and back into communities.	SDG 16 (Peace, Justice, and Strong Institutions)

Table 3. Parameter Descriptions and Values from Literature

Parameter	Description	Typical Value / Range	Source / Notes
π	Recruitment rate into the susceptible pool	5–20 persons/year	Demographic data (Lawal et al., 2023)
μ	Natural exit (mortality/attrition) rate	0.01–0.05 per year	Demographic assumption
β	Effective contact (recruitment) rate of bandits	0.2–0.6	(Aniyam et al., 2018)
ϕ	External risk pressure (structural insecurity)	0.01–0.05	(Momo et al., 2023)
σ	Progression rate from exposed to active bandits	0.1–0.4	(Lawal et al., 2023)
e_2	Effectiveness of education/employment control (u_2)	0–1	Control effectiveness parameter
ρ	Recruitment rate of informants by bandits	0.05–0.2	(Aniyam et al., 2018)
r_1	Effectiveness of security/justice control (u_1)	0–1	Control effectiveness parameter
η	Rate of bandit recruitment into informants	0.1–0.3	(Momo et al., 2023)
τ	Reintegration rate of informants back to the susceptible through jobs/education	0.05–0.2	(Lawal et al., 2023)
δ_1	Rehabilitation rate due to reintegration programs (u_3)	0.1–0.3	(Momo et al., 2023)
δ_2	Bandit removal through policing and justice (u_1)	0.1–0.4	(Aniyam et al., 2018)
ω	Natural disengagement rate (voluntary exit)	0.01–0.05	Assumed small
α_B	Bandit-related mortality or permanent exit	0.05–0.1	(Lawal et al., 2023)
γ	Rate of return from rehabilitation to susceptible	0.05–0.2	Reintegration reports

Assumptions of the Model

1. Total population:
   $N(t) = S + E + B + I + R$
2. Recruitment increases with B (mass action). Baseline deprivation adds ϕ.
3. Education and jobs reduce progression from $E \to B$
4. Security reduces $I \to B$ and increases $B \to R$
5. Rehabilitation increases returned flows $R, I \to S$
6. All classes experience natural exit μ. Bandits face extra mortality α_B
7. Controls are bounded, measurements, and carry convex cost.

Force of risk and controlled Transitions

In this model, the *force of risk* reflects how likely vulnerable students are to be drawn into banditry. That risk grows when young people are exposed to peers already involved, pressured by poverty, or left without strong institutional protection. It works much like an "infection rate" in disease models; the higher the exposure, the greater the chance of recruitment.

$$\Lambda(t) = \frac{\beta B}{N} + \phi$$
$$E \to B : \sigma(1 - e_2 u_2),$$
$$B \to R : \delta_1 u_3 + \delta_2 u_1 + \omega,$$
$$I \to B : \rho(1 - r_1 u_1),$$
$$I \to S : \tau u_2.$$
$$R \to S : \gamma u_3$$

MODEL EQUATIOMS

The susceptible group (S) represents people in society who are at risk of being drawn into banditry because of poverty, unemployment, and weak support systems. New individuals enter this class through natural recruitment at rate π, while some who had previously recovered may lose resilience and return at rate $\gamma u_3 R$, where u_3 reflects reintegration follow-up. People leave the susceptible class when they are recruited into exposure through the "force of infection" ΛS, which captures the combined pressure of propaganda, peer influence, and socio-economic hardship. Natural death also reduces this group at rate μS, while preventive interventions such as education and empowerment allow some to avoid slipping into exposure, represented by the feedback $\tau u_2 I$, where u_2 models of socio-economic empowerment.

$$\frac{dS}{dt} = \pi - \Lambda S - \mu S + \tau u_2 I + \gamma u_3 R \quad \textbf{(1)}$$

The exposed class (E) includes individuals who are influenced by bandits but not yet fully active. They are recruited from the susceptible pool by the force of infection ΛS. Without intervention, many progress into becoming recruits at a rate $\sigma(1 - e_2 u_2) E$, where σ measures the speed of recruitment, and $e_2 u_2$ reflects the effectiveness of prevention and counter-radicalization programs. Death also occurs naturally in this group at rate μE. This compartment shows the delicate stage where timely community sensitization and peacebuilding can stop people from progressing further into banditry.

$$\frac{dE}{dt} = \Lambda S - \sigma(1 - e_2 u_2) E - \mu E, \quad (2)$$

The bandit recruit class (B) captures those who have taken a step further by joining bandit groups, either willingly or by coercion. They are fueled by exposed individuals transitioning at a rate $\sigma(1-e_2u_2)E$, and by relapse from active bandits at rate $\rho(1-r_1u_1)I$, where ρ captures re-radicalization pressure and r_1u_1 reflects the strength of legal reforms or justice interventions. Recruits can move out of this stage through rehabilitation at rates δ_2u_3B and δ_2u_1B, where δ_1 and δ_2 measure reintegration effectiveness under interventions u_3 and u_1. Others are lost through natural clearance at rate ωB, natural death at μB, or direct removal due to violent encounters at $\alpha_B B$.

$$\frac{dB}{dt}=\sigma(1-e_2u_2)E+\rho(1-r_1u_1)I-(\delta_1u_3+\delta_2u_1+\omega+\mu+\alpha_B)B, \quad (3)$$

The active bandit class (I) is the most dangerous group, made up of individuals who fully participate in raids, kidnappings, and violence. They are generated mainly from the recruit pool at rate ηB, where η reflects the proportion of recruits that become fully operational. Some active bandits relapse back into recruitment through re-radicalization pressure at $\rho(1-r_1u_1)I$, while others exit through treatment-like interventions such as targeted security operations or peace dialogues, captured by $\tau u_2 I$, where u_2 represents socio-economic stabilization efforts. This group also shrinks through natural death at rate μI.

$$\frac{dI}{dt}=\eta B-\rho(1-r_1u_1)I-(\tau u_2+\mu)I, \quad (4)$$

Finally, the recovered group (R) represents individuals who successfully disengage from banditry and are reintegrated into society. They enter from the recruit class through rehabilitation at rates δ_1u_3B and δ_2u_1B, or by natural clearance at ωB. This pool also includes those redirected from active participation through interventions. However, recovery is not always permanent; some may relapse into susceptibility at rate $\gamma u_3 R$, if social and economic support fails. Natural death also reduces this class at rate μR. This group illustrates the long-term goal of rehabilitation and reintegration programs aligned with SDGs, particularly by linking peace with sustainable livelihoods.

$$\frac{dR}{dt}=(\delta_1u_3+\delta_2u_1+\omega)B-(\gamma u_3+\mu)R \quad (5)$$

Thus, see the model equations below

$$\left.\begin{aligned}
\frac{dS}{dt} &= \pi - \Lambda S - \mu S + \tau u_2 I + \gamma u_3 R \\
\frac{dE}{dt} &= \Lambda S - \sigma\left(1-e_2u_2\right)E - \mu E, \\
\frac{dB}{dt} &= \sigma\left(1-e_2u_2\right)E + \rho\left(1-r_1u_1\right)I - \left(\delta_1u_3 + \delta_2u_1 + \omega + \mu + \alpha_B\right)B, \\
\frac{dI}{dt} &= \eta B - \rho\left(1-r_1u_1\right)I - \left(\tau u_2 + \mu\right)I, \\
\frac{dR}{dt} &= \left(\delta_1u_3 + \delta_2u_1 + \omega\right)B - \left(\gamma u_3 + \mu\right)R
\end{aligned}\right\} \quad \textbf{(6)}$$

Positivity and boundedness

$$\left.\begin{aligned}
\frac{dS}{dt} &= \pi - \Lambda S - \mu S + \tau u_2 I + \gamma u_3 R, \\
\frac{dE}{dt} &= \Lambda S - \sigma\left(1-e_2u_2\right)E - \mu E, \\
\frac{dB}{dt} &= \sigma\left(1-e_2u_2\right)E + \rho\left(1-r_1u_1\right)I - \left(\delta_1u_3 + \delta_2u_1 + \omega + \mu + \alpha_B\right)B, \\
\frac{dI}{dt} &= \eta B - \rho\left(1-r_1u_1\right)I - \left(\tau u_2 + \mu\right)I, \\
\frac{dR}{dt} &= \left(\delta_1u_3 + \delta_2u_1 + \omega\right)B - \left(\gamma u_3 + \mu\right)R
\end{aligned}\right\}$$

With

$$\Lambda(t) = \frac{\beta B}{N} + \phi\,,\ N = S + E + B + I + R \ \ and\ \ 0 \le u_j(t) \le 1. \qquad (7)$$

Positivity

Assume that each state remains nonnegative for all $t \ge 0$.

Boundary checks:

$$\left.\begin{aligned}
\left.\frac{dS}{dt}\right|_{S=0} &= \pi + \tau u_2 I + \gamma u_3 R \geq 0, \\
\left.\frac{dE}{dt}\right|_{E=0} &= \Lambda S \geq 0, \\
\left.\frac{dB}{dt}\right|_{B=0} &= \sigma\left(1-e_2u_2\right)E + \rho\left(1-r_1u_1\right)I \geq 0, \\
\left.\frac{dI}{dt}\right|_{I=0} &= \eta B \geq 0, \\
\left.\frac{dR}{dt}\right|_{R=0} &= \left(\delta_1 u_3 + \delta_2 u_1 + \omega\right)B \geq 0.
\end{aligned}\right\} \quad \textbf{(8)}$$

Each derivative at the nonnegative boundary is nonnegative. The nonnegative orthant is forward invariant. Standard ODE comparison results apply because the right-hand sides are locally Lipschitz in the states and measurable bounded in the controls (Lenhart & Workman, 2007). Therefore, solutions with nonnegative initial data stay nonnegative.

Boundedness and invariant set

Sum the Model equations:

$$\frac{dN}{dt} = \pi - \mu N - \alpha_B B, \tag{9}$$

All internal transfers cancel. The remaining sink $-\alpha_B B \leq 0$ gives

$$\frac{dN}{dt} \leq \pi - \mu N, \tag{10}$$

Solve the scalar inequality by comparison with $\dot{y} = \pi - \mu y$

$$N(t) \leq \frac{\pi}{\mu} + \left(N(0) - \frac{\pi}{\mu}\right)e^{-\mu t}. \tag{11}$$

Thus,

$$0 \leq N(t) \leq \max\left\{N(0), \frac{\pi}{\mu}\right\} \quad for\ all\ t \geq 0. \tag{12}$$

Define the feasible region $\Gamma = \left\{(S,E,B,I,R) \in \mathbb{R}^5_{\geq 0} : N \leq \frac{\pi}{\mu}\right\}.$ (13)

The positivity and the bound on N , the set Γ is positively invariant and attracts all trajectories with $N(0) \le \frac{\pi}{\mu}$. Existence and uniqueness hold globally because solutions never blow up and the vector field is locally Lipschitz on Γ (Lenhart & Workman).

Banditry-Free and Banditry Endemic Equilibria

Banditry -Free equilibrium (DFE): $E_0 = \left(\frac{\pi}{\mu}, 0, 0, 0, 0\right)$ (14)

Banditry Endemic Equilibrium

Define

$$\left.\begin{aligned} k_l &= \frac{\eta}{\rho(1-r_1u_1)+\tau u_2+\mu}, \quad k_R = \frac{\delta_1 u_3+\delta_2 u_1+\omega}{\gamma u_3+\mu}, \\ k_E &= \frac{(\delta_1 u_3+\delta_2 u_1+\omega+\mu+\alpha_B)-\rho(1-r_1u_1)k_l}{\sigma(1-e_2u_2)} \end{aligned}\right\} \quad (15)$$

Then the Banditry Endemic Equilibrium is

$$(S^*, E^*, B^*, I^*, R^*) = \left(N^* - (1+k_E+k_l+k_R)B^*, k_E B^*, k_l B^*, k_R B^*\right), \quad (16)$$

Where $N^* = S^* + E^* + B^* + I^* + R^* \; and \; B^* > 0$ .The positive solution of the equilibrium balance implied by the system is (exists when $\Re_0 > 1$).

Basic Reproduction Number $\Re_0$

Using the next-generation matrix method (Diekman et al., 1990), we obtain:

$$\Re_0 = \frac{\beta\sigma(1-e_2u_2)[\tau u_3+\mu+\rho(1-r_1u_1)]}{\left[\sigma(1-e_2u_2)+\mu\right]\left[(\delta_1u_3+\delta_2u_1+\omega+\alpha_B)(\tau u_2+\mu+\rho(1-r_1u_1))-\rho(1-r_1u_1)\eta\right]} \quad (17)$$

When $\Re_0 < 1$; the banditry-free equilibrium is locally asymptotically stable, otherwise, the endemic equilibrium may exist.

Objective functional

We aim to minimize the cost functional

$$J\left(u_1,u_2,u_3\right)=\int_0^T A_1B+A_2I+\frac{1}{2}\left(C_1u_1^2+C_2u_2^2+C_3u_3^2\right)dt \tag{18}$$

Where A_1, A_2 penalize banditry and influence, and C_1, C_2, C_3 are cost weights.

Admissible controls

$$0\le u_j\left(t\right)\le 1,\quad j=1,2,3. \tag{19}$$

Hamilton

Introduce adjoint variables $\lambda_1,\ldots,\lambda_5$. Hamiltonian is

$$\left.\begin{aligned}H=\;&A_1B+A_2I+\frac{1}{2}\left(C_1u_1^2+C_2u_2^2+C_3u_3^2\right)\\&+\lambda_1\left(\pi-\Lambda S-\mu S+\tau u_2I+\gamma u_3R\right)\\&+\lambda_2\left(\Lambda S-\sigma\left(1-e_2u_2\right)E-\mu E\right)\\&+\lambda_3\left(\sigma\left(1-e_2u_2\right)E+\rho\left(1-r_1u_1\right)I-\left(\delta_1u_3+\delta_2u_1+\omega+\mu+\alpha_B\right)B\right)\\&+\lambda_4\left(\eta B-\rho\left(1-r_1u_1\right)I-\left(\tau u_2+\mu\right)I\right)\\&+\lambda_5\left(\left(\delta_1u_3+\delta_2u_1+\omega\right)B-\left(\gamma u_3+\mu\right)R\right)\end{aligned}\right\} \tag{20}$$

Adjoint system

With transversality $\lambda_i(T)=0$ we obtain:

$$\left.\begin{aligned}
\dot{\lambda}_1 &= \lambda_1(\Lambda+\mu)-\lambda_2\Lambda+(\lambda_1-\lambda_2)\frac{S\beta B}{N^2},\\
\dot{\lambda}_2 &= \lambda_2\left(\sigma(1-e_2u_2)+\mu\right)-\lambda_3\sigma(1-e_2u_2)+(\lambda_1-\lambda_3)\frac{S\beta B}{N^2},\\
\dot{\lambda}_3 &= -A_1+(\lambda_1-\lambda_2)\frac{S\beta(N-B)}{N^2}+\lambda_3(\delta_1u_3+\delta_2u_1+\omega+\mu+\alpha_B)-\lambda_4\eta+\lambda_5(\delta_1u_3+\delta_2u_1+\omega),\\
\dot{\lambda}_4 &= -A_1-\lambda_1\tau u_2-\lambda_3\rho(1-r_1u_1)+\lambda_4\left(\rho(1-r_1u_1)+\tau u_2+\mu\right)+(\lambda_1-\lambda_2)\frac{S\beta B}{N^2},\\
\dot{\lambda}_5 &= -\lambda_1\gamma u_3+\lambda_5(\gamma u_3-\mu)+(\lambda_1-\lambda_2)\frac{S\beta B}{N^2}
\end{aligned}\right\} \quad (21)$$

Characterization of controls

For each control u_k, optimality requires:

$$\left.\begin{aligned}
u_1^*(t) &= \min\left\{1,\max\left\{0,\frac{\rho r_1 I(\lambda_4-\lambda_3)+\delta_2 B(\lambda_5-\lambda_3)}{C_1}\right\}\right\},\\
u_2^*(t) &= \min\left\{1,\max\left\{0,\frac{\tau I(\lambda_1-\lambda_4)+\sigma e_2 E\lambda_2}{C_2}\right\}\right\},\\
u_3^*(t) &= \min\left\{1,\max\left\{0,\frac{\gamma R(\lambda_1-\lambda_5)+\delta_1 B(\lambda_5-\lambda_3)}{C_3}\right\}\right\}
\end{aligned}\right\} \quad (22)$$

Transversality $\lambda_i(T)=0,\ \ i=1,\ldots,5.$ (23)

NUMERICAL SIMULATIONS

In the numerical simulation, the forward-backward sweep method was applied to find the optimal strategies. The process begins with trial values for the three controls, after which the state equations are solved step by step, moving forward in time. The adjoint equations are then worked out in reverse, starting from the end of the time horizon and working backward to the beginning. Using both sets of solutions, the control functions are updated and adjusted to remain within the allowable range. This cycle is repeated until the results stabilize. Once convergence is reached, the time paths of the state variables and controls are presented in figures, illustrating how justice reforms (u_1), education and livelihood programs (u_2), and rehabilitation measures (u_3) can influence the dynamics of banditry.

Control Strategies

1. Strategy A (Justice/Legal Reforms + Education and Jobs: u_1+u_2).
2. Strategy B (Justice/Legal Reforms + Rehabilitation and Reintegration: u_1+u_3).
3. Strategy C (Education and Jobs + Rehabilitation and Reintegration: u_2+u_3)
4. Strategy D (Comprehensive Approach: $u_1+u_2+u_3$)

Strategy A (Justice/Legal Reforms + Education and Jobs: u_1+u_2)
Focuses on strengthening justice systems while providing education and employment opportunities. This reduces both re-recruitment into banditry and the vulnerability of at-risk youth. See figure 1 to 5.

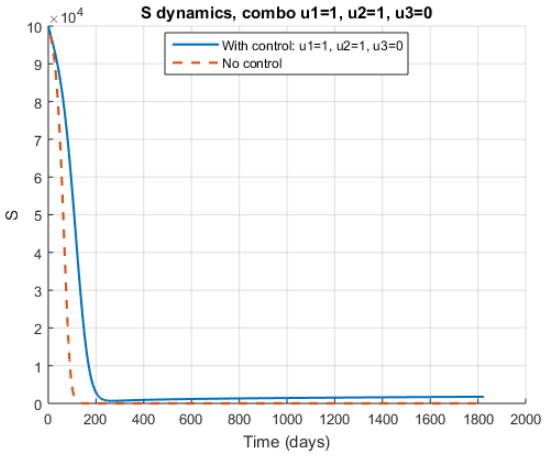


Figure 1 – Susceptible (S) under u_1+u_2. The number of people who remain outside banditry rises above the baseline. Security measures (u_1) reduce pressure from informants and bandits, while education and jobs (u_2) prevent young people from progressing into crime. Implication: Sustained investment in security and livelihoods enlarges the pool of safe citizens, reducing the chances of new recruits

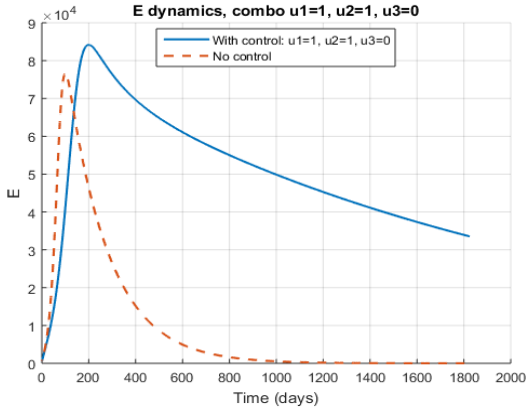
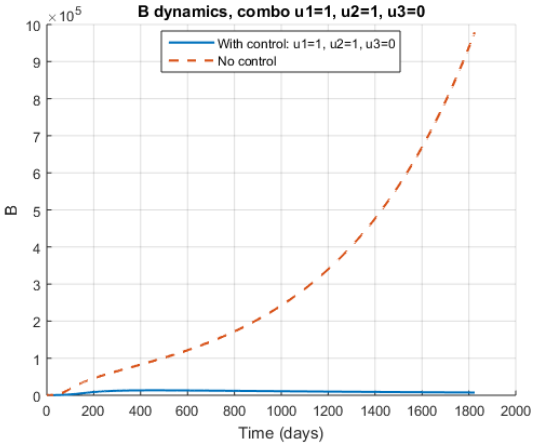


Figure 2 – Exposed (E) under u_1+u_2. Exposure to banditry declines much faster compared to the baseline. Fewer recruits progress into banditry because u2 blocks the E→B transition, and u1 reduces the "pull" effect of existing criminals. Implication: Cutting off the pipeline of potential recruits early is one of the most effective ways to weaken banditry long-term.



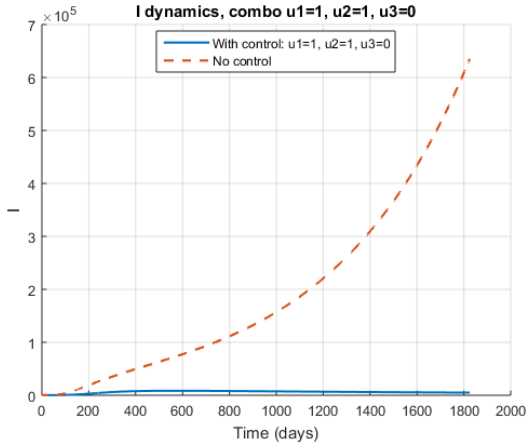


Figure 3 – Bandits (B) under u1+u2.The number of active bandits falls steeply. The combined effects of limited inflow (u2) and reduced recycling from informants (u1) shrink the size of the core criminal group. Implication: Security plus job creation can drastically weaken bandit groups, making them easier to dismantle.

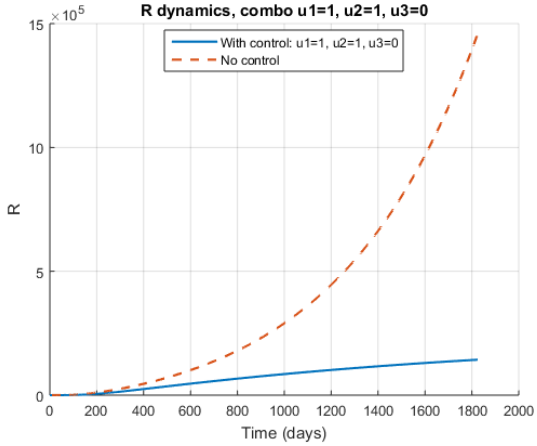


Figure 5 – Rehabilitation (R) under u1+u2
Rehabilitation numbers rise. Bandits exit to R through enforcement (u1) and limited natural exits (ω), while fewer new criminals join. Implication: More reintegration opportunities emerge, showing how security can be paired with reintegration programs to absorb ex-bandits back into society.

Figure 4 – Informants (I) under u1+u2. Informant activity drops below the baseline. Since banditry declines, fewer people are motivated to spy or collaborate, and some are returned to normal life through u2. Implication: Weakening bandit groups reduces their intelligence networks, disrupting coordination and supply chains.

Strategy B (Justice/Legal Reforms + Rehabilitation and Reintegration: u1+u3)

Combines deterrence through legal reforms with rehabilitation efforts, ensuring that former bandits are reintegrated into society and are less likely to relapse. See figures 6 to 10

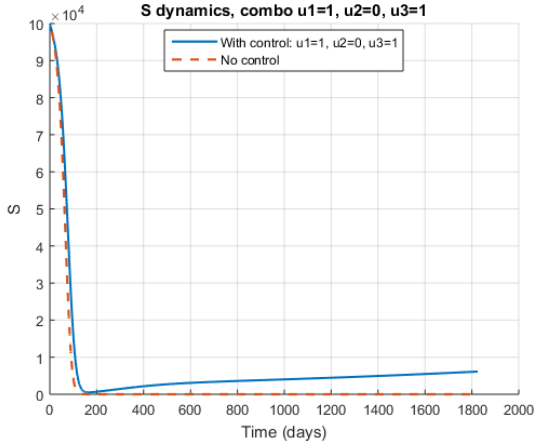


Figure 6 – Susceptible (S) under u1+u3. Safe citizens increase strongly. Rehabilitation (u3) returns people

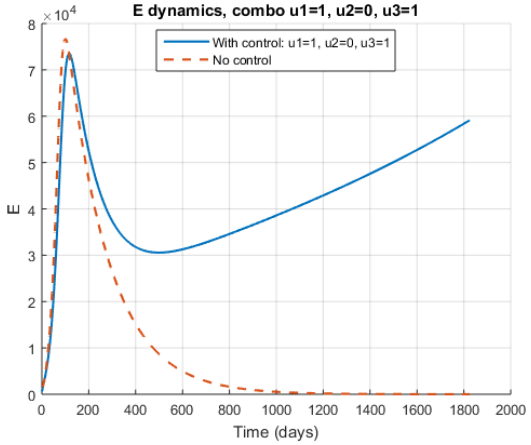


Figure 7 – Exposed (E) under u1+u3. Exposures drop. Even though U2 is absent, u1 and u3 still reduce crime

from crime back into society, while u1 suppresses informant-driven recruitment. Implication: A justice-plus-rehabilitation mix rebuilds trust and strengthens communities.

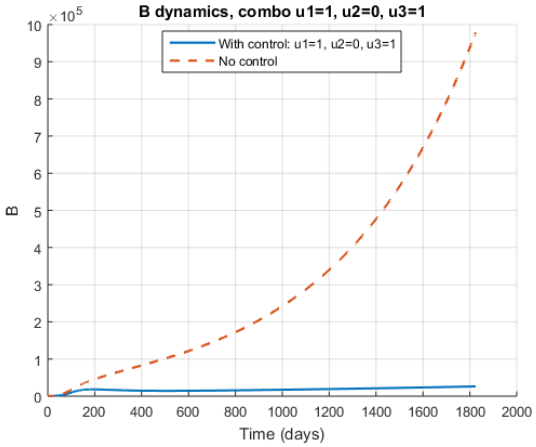


Figure 8 – Bandits (B) under u1+u3Bandit numbers decline sharply. u1 blocks inflow from informants, while u3 drains bandits into rehabilitation. Implication: Combining force with forgiveness simultaneously shrinks criminal groups and offers peaceful exits.

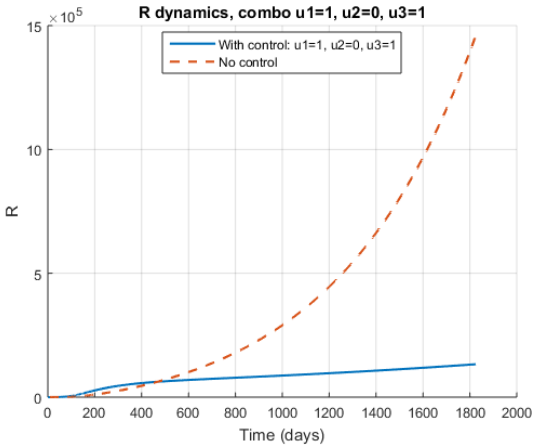


Figure 10 – Rehabilitation (R) under u1+u3. Rehabilitation rises more strongly than in the u1+u2 mix, because u3 actively pushes bandits into reintegration. Implication: This strategy creates more opportunities for deradicalization, reconciliation, and the return of ex-combatants.

pressure by lowering the number of active bandits, which in turn reduces the risk of exposure. Implication: Less crime on the streets naturally reduces the temptation or coercion for youth to join.

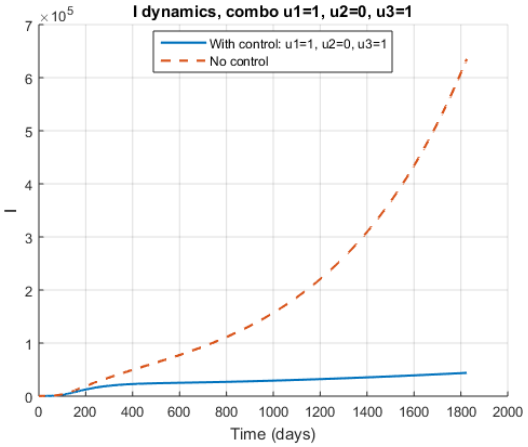


Figure 9 – Informants (I) under. u1+u3Informants fall as bandits are dismantled. Without a large B population, the value of serving as an informant diminishes. Implication: Community cooperation with bandit groups weakens, making them easier to monitor and suppress.

Strategy C (Education and Jobs + Rehabilitation and Reintegration: u2+u3) Prevents new recruitment by addressing socio-economic needs while also supporting the reintegration of ex-bandits, thus reducing community-level vulnerability. See figures 11 to 15.

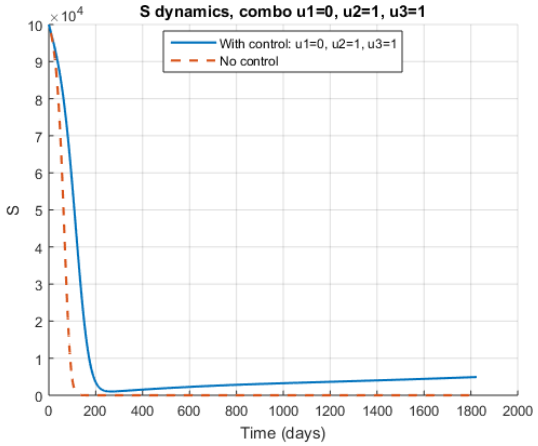


Figure 11 – Susceptible (S) under u2+u3.The susceptible population rises gradually. Education and jobs (u2) reduce entry into crime, while rehabilitation (u3) cycles people back into normal life. Implication: Social policies alone (without strong security enforcement) still grow safe communities, but more slowly.

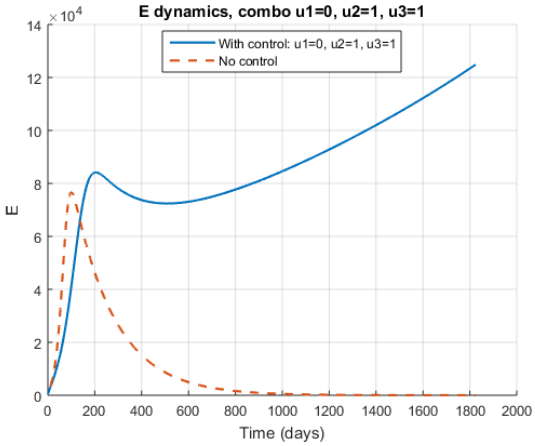


Figure 12 – Exposed (E) under u2+u3
Exposure falls strongly because fewer people progress to crime, and fewer bandits remain active to recruit. Implication: Youth empowerment plus rehabilitation cuts both the "push" and "pull" of criminality.

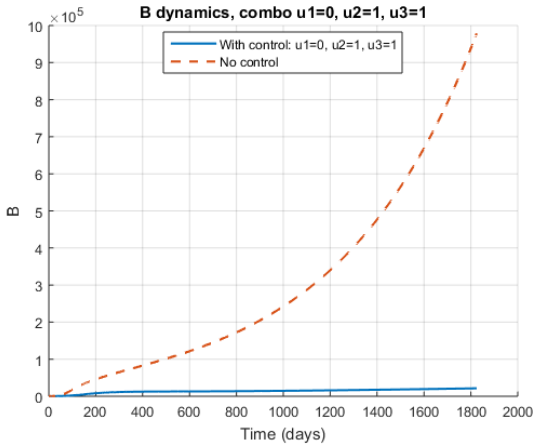


Figure 13 – Bandits (B) under u2+u3. Bandit numbers shrink steadily, though not as rapidly as when u1 is included. Implication: Non-violent policies alone can reduce crime but may take longer and require sustained investment.

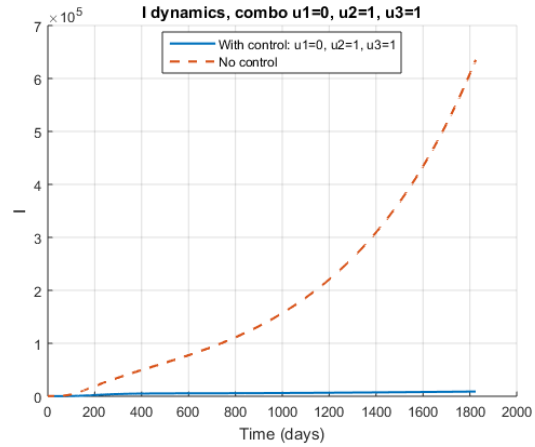


Figure 14 – Informants (I) under u2+u3. Informant networks weaken because B shrinks, though more slowly than in mixes with u1. Implication: Social measures undermine criminal intelligence networks over time, though they lack the immediate impact of enforcement

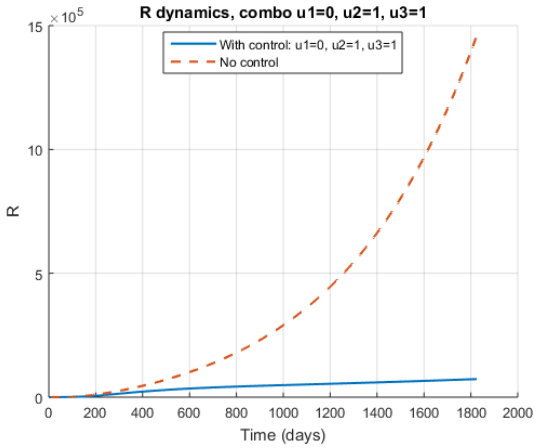


Figure 15 – Rehabilitation (R) under u2+u3. Rehabilitation rises steadily. Bandits are pulled into peaceful reintegration (u3), while u2 reduces inflow to replace them. Implication: A positive cycle of reform builds momentum, shifting norms away from violence.

Strategy D (Comprehensive Approach: u1+u2+u3)

Integrates all three levers: justice reforms, education, jobs, and rehabilitation. This holistic package addresses prevention, deterrence, and reintegration simultaneously, producing the strongest long-term impact. See figures 16 to 20.

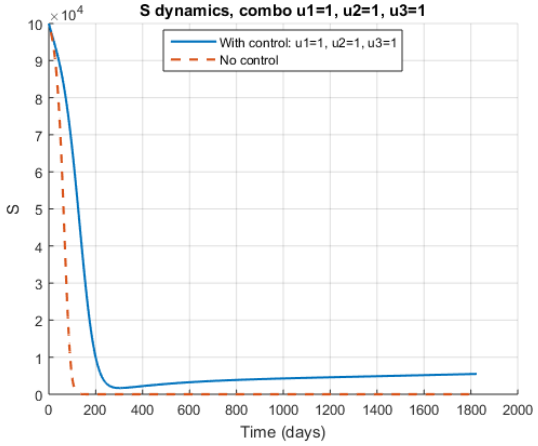


Figure 16 – Susceptible (S) under u1+u2+u3. This combination produces the largest pool of safe citizens. All levers act together: fewer exposures, faster exits, and strong returns to normal life. Implication: A comprehensive approach is the most effective way to rebuild stable, resilient communities.

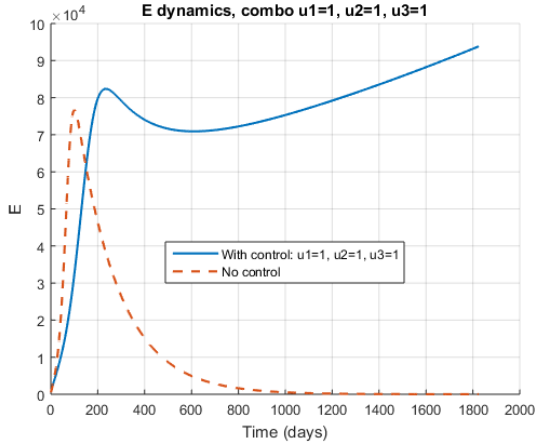


Figure 17 – Exposed (E) under u1+u2+u3. Exposure is minimized. Every pathway into crime is weakened, and every pathway out is strengthened. Implication: This mix essentially closes the recruitment pipeline, cutting off the lifeblood of banditry.

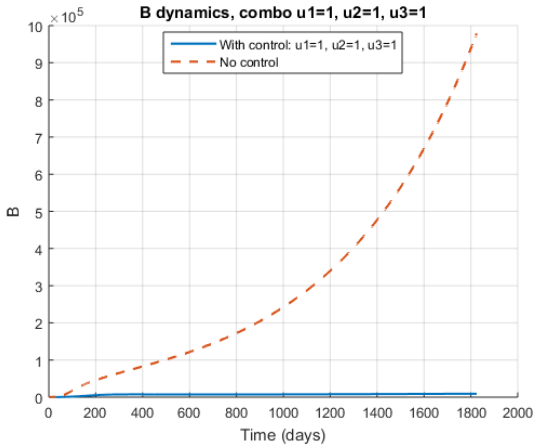


Figure 18 – Bandits (B) under u1+u2+u3 Banditry collapses fastest and deepest under the full mix. Implication: Only integrated strategies (security, livelihoods, and rehabilitation together) can realistically break the cycle of violence.

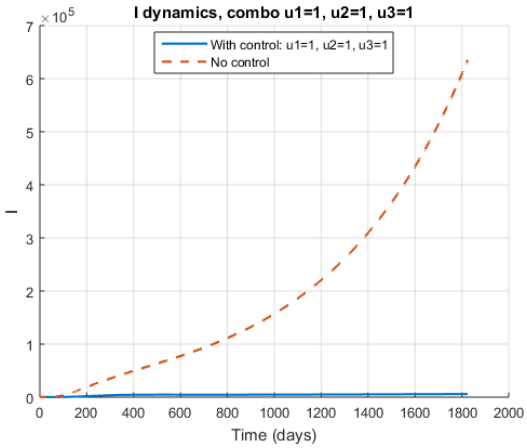


Figure 19 – Informants (I) under u1+u2+u3 Informants vanish more quickly than in any other mix. Implication: As the main group disintegrates, support structures disappear, isolating remaining criminals.

.

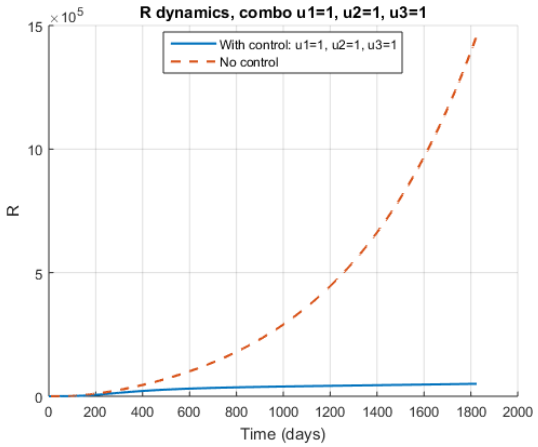


Figure 20 – Rehabilitation (R) under u1+u2+u3 Rehabilitation peaks, then stabilizes as fewer bandits remain. This reflects a healthy cycle: criminals reintegrate while society strengthens. Implication: Full strategies don't just fight crime—they create long-term peace by reintegrating ex-bandits.

Summary of B (t) across mixes

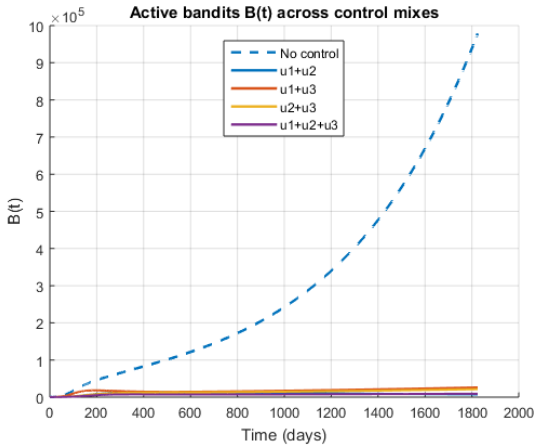


Figure 21 – Summary of B(t) across mixes. This comparative plot shows the clear hierarchy: full controls (u1+u2+u3) are most effective, then pairs, then no control. Implication: To decisively reduce banditry, governments and communities must combine security, social opportunities, and reintegration, not rely on single interventions

Discussions

The analysis confirms that the most powerful pathway to reducing banditry lies in the integrated application of all three interventions: security and justice (u_1), education and jobs (u_2), and rehabilitation and reintegration (u_3). Taken together, these interventions address the three pillars of conflict dynamics:

1. Prevention of recruitment – ensuring that vulnerable youth, often targeted because of poverty, unemployment, and marginalization, are provided with opportunities that divert them from criminal networks.
2. Suppression of active criminality – using justice and law enforcement mechanisms to dismantle existing groups, disrupt command-and-control networks, and weaken the support base of informants and collaborators.
3. Reintegration and resilience-building – providing ex-bandits with credible and safe pathways back into society, thereby closing the cycle of violence and ensuring that former offenders become part of the peacebuilding process rather than sources of relapse.

This "all-three" approach essentially operates as a systemic intervention, recognizing that banditry is not a linear phenomenon but rather a self-reinforcing cycle of violence, poverty, and mistrust. By targeting all stages of the cycle simultaneously, the approach does not only reduce the number of active bandits (B) but also prevents new exposure (E) and breaks down the role of informants (I), while expanding the share of the susceptible (S)

population that remains peaceful and engaged in productive livelihoods. This integrated framework reflects the multidimensional nature of insecurity in West Africa, where socio-economic deprivation, weak governance, and criminal opportunism overlap.

However, in real-world governance settings, resource constraints are inevitable. Governments often face competing priorities such as health, infrastructure, and education, which limit their ability to deploy all interventions at once. In such scenarios, prioritization becomes critical. The analysis indicates that among the two-strategy combinations, the most impactful is the pairing of security and justice with education and jobs ($u_1 + u_2$). This combination ensures immediate results through enforcement while simultaneously addressing the structural drivers of recruitment. Enforcement alone may offer a short-term reduction in attacks, but without job creation and education, the recruitment pipeline remains open, leading to a resurgence of violence. Likewise, social interventions without security would be undermined by ongoing violence that deters communities from accessing education and livelihoods. Thus, $u_1 + u_2$ emerges as the most balanced and practical option, striking a midpoint between immediate suppression and long-term prevention.

The role of rehabilitation and reintegration (u_3), though sometimes underfunded, should not be underestimated. In contexts where thousands of youth have already been recruited, punitive security-only measures risk further alienating communities and may inadvertently fuel cycles of revenge. Reintegration, when combined with preventive and suppressive measures, rebuilds trust between communities and the state, reduces stigmatization of ex-combatants, and gradually normalizes peace. This explains why the $u_2 + u_3$ combination, although slower in producing visible short-term gains, results in a deeper, more sustainable transformation in the long run. Rehabilitation also fosters community resilience, reducing the likelihood of relapse into criminality by providing ex-bandits with social networks, vocational training, and psycho-social support that anchor them in civilian life.

From a modeling perspective, the optimal controls highlight the complementary nature of the interventions. Security (u_1) directly reduces the effective reproduction number of banditry, ensuring that each bandit is unable to recruit or sustain replacements. Education and jobs (u_2) weaken recruitment intensity and shift potential informants away from risky roles, cutting off the flow of new entrants into crime. Rehabilitation and reintegration (u_3) function as a restorative measure, reinvesting human capital back into society and ensuring that peace dividends are widely shared. Collectively, these dynamics illustrate why fragmented approaches—whether solely militarized crackdowns or purely social responses—are insufficient.

The findings also have comparative significance. In northern Mali, Niger, and Mauritania, where similar dynamics of youth vulnerability and weak governance exist, governments have often leaned heavily on military approaches, with limited success. By contrast, where socio-economic interventions and reconciliation programs were combined with enforcement, as seen in parts of the

Niger Delta Amnesty Programme in Nigeria, violence declined more substantially, though sustainability was undermined by governance failures and corruption. This underscores the importance of designing balanced, transparent, and accountable interventions that are context-sensitive and community-driven.

CONCLUSION

This study clearly demonstrates that banditry cannot be effectively addressed by a single-dimensional approach. The dynamics revealed through the mathematical modeling emphasize that the cycle of recruitment, violence, and reintegration must be disrupted at multiple entry points simultaneously. The integrated application of security and justice (u_1), education and employment (u_2), and rehabilitation and reintegration (u_3) offers the strongest and most sustainable pathway. When applied together, these interventions:

- Block recruitment pipelines for vulnerable youth,
- Dismantle active networks through immediate enforcement, and
- Create safe, credible exit strategies that allow ex-bandits to return to normal civilian life.

The policy implication is unequivocal: *no single strategy—whether purely militarized or purely socio-economic—will suffice on its own.* Fragmented approaches risk either temporary gains that quickly unravel, or long-term programs that are continuously undermined by ongoing violence. Instead, governments must design hybrid strategies that deliberately link enforcement with socio-economic transformation.

Nevertheless, in contexts of limited resources, the dual combination of security and justice with education and jobs ($u_1 + u_2$) emerges as the most practical and impactful option. It balances short-term enforcement with long-term prevention, delivering immediate reductions in violence while building the foundations for sustainable peace through livelihood opportunities. The alternative pairing of education and jobs with rehabilitation ($u_2 + u_3$), while slower to show results, lays a stronger foundation for deep structural change, especially where governments commit to sustained reintegration programs. Thus, policymakers must weigh short-term urgency against long-term resilience when designing intervention mixes.

Importantly, aligning these interventions with the Sustainable Development Goals (SDGs) strengthens both their policy legitimacy and their implementation framework. SDG 4 (quality education) ensures that preventive measures are grounded in accessible learning opportunities; SDG 8 (decent work and economic growth) underscores the necessity of livelihood creation to divert youth from criminality; and SDG 16 (peace, justice, and strong institutions) anchors enforcement within the rule of law, ensuring accountability and fairness. By situating banditry interventions within the SDG framework, governments in Nigeria and the wider West African region can pursue strategies that are not only effective in the short term but also transformative in addressing the deeper structural drivers of insecurity.

In summary, the all-three strategy remains the gold standard for combating banditry, combining immediate, medium-term, and long-term gains into a coherent and sustainable framework. Where this is not feasible, the dual strategy of security and justice with education and jobs offers the best

compromise between effectiveness and feasibility. Future policies should therefore emphasize integrated, resource-sensitive approaches, guided by both mathematical modeling insights and real-world governance contexts.

Acknowledgment

The authors gratefully acknowledge the financial support of the Tertiary Education Trust Fund (TETFund). This sponsorship was instrumental in executing the research project: "*Mathematical Modeling of Banditry and Its Optimal Control Using Sustainable Development Goals (SDGs) as Intervention Strategies: Mathematical Pathways to Peace.*" The support of TETFund underscores its commitment to advancing scholarship and addressing pressing societal challenges for the collective good of humanity.

References

Abdullahi, H. I. (2019). *Banditry and rural insecurity in Niger Republic of Niger: Socioeconomic implications*. Niamey: Centre for Peace Studies.

Abdulrahman, A., Bello, M., & Lawal, Y. (2024). Epidemic-inspired mathematical modeling of rural banditry in Northern Nigeria. *Journal of Applied Mathematical Analysis, 62*(1), 77–95. https://doi.org/10.1016/j.jama.2024.01.006

Accord. (2022). *Resource competition and rural violence in Mauritania*. Geneva: African Centre for Conflict Resolution.

Adeoye, A. (2018). Banditry and governance failures in Nigeria. *African Journal of Criminology, 12*(2), 44–59.

Aniyam, A., Okeke, E., & Ibrahim, M. (2018). A mathematical model of kidnapping: Rehabilitation and deradicalization dynamics. *Journal of Applied Mathematics, 34*(1), 101–117.

Bagu, C., & Smith, D. (2017). *Communities under siege: Rural banditry in North-West Nigeria*. Abuja: Centre for Democracy and Development.

Damilola, O. (2025). Psychological effects of banditry on students in Nigerian tertiary institutions. *Nigerian Journal of Education and Development Studies, 15*(2), 88–104.

Global Security. (2022). *Nigeria: Banditry and rural insecurity*. Retrieved from https://www.globalsecurity.org

Global Terrorism Index. (2018). *Measuring the impact of terrorism*. Sydney: Institute for Economics and Peace.

Hunter, J. (2013). Armed robbery and organized crime in Nigeria. *Journal of African Security Studies, 8*(3), 66–82.

Idowu, A., & Eze, N. (2024). Banditry and institutional development in Nigerian universities: An empirical analysis. *African Journal of Higher Education Research, 19*(1), 34–56.

Lawal, A., Musa, A., & Danladi, I. (2023). Optimal control of armed banditry in Nigeria: A mathematical modeling approach. *Journal of Security and Development Studies, 11*(2), 55–74.

Mohammed, A., Umar, A., Olaleye, B., Pelemo, S., Ahmed, Y., Mohammed, T., & Umar, M. (2021). Banditry and human security in Northern Nigeria. *Nigerian Journal of Peace and Conflict Studies, 9*(1), 1–19.

Momo, K., Abdulahi, Y., & Bello, H. (2023). Modeling the dynamics of rural banditry: A systems approach. *Mathematical Social Sciences, 119*, 23–39. https://doi.org/10.1016/j.mathsocsci.2023.06.004

Mustapha, A. (2019). *The politics of rural insecurity in Zamfara State*. Kaduna: Arewa Research Foundation.

Neack, L. (2017). *Elusive security: States first, people last*. Lanham, MD: Rowman & Littlefield.

Ogbonnaya, U. (2020). Illegal mining, governance, and rural insecurity in Nigeria. *African Studies Review, 63*(3), 499–520.

Okoli, A. C., & Ugwu, C. (2019). Of marauders and brigands: The scourge of rural banditry in Nigeria. *African Journal on Conflict Resolution, 19*(2), 91–120. https://doi.org/10.4314/ajcr.v19i2.5

Okoye, P., Nwachukwu, J., & Suleiman, A. (2020). Mathematical modeling of terrorism dynamics for policy insights. *International Journal of Mathematics and Computation, 30*(3), 75–89.

Okoye, C., & Nwaka-Nwandu, C. (2023). Banditry and institutional performance in Nigerian tertiary education: A probit regression analysis. *Journal of Educational Policy and Development Studies, 18*(4), 222–240.

Rosenje, B., & Adeniyi, K. (2021). Rural banditry and national security challenges in Nigeria. *Journal of Social and Political Research, 7*(1), 34–52.

Rufa'i, A. (2021). *The political economy of banditry and illegal mining in Zamfara State*. Kano: Bayero University Press.

Shockden, T. (2023). *School closures and child trauma in conflict-prone regions of Nigeria*. Lagos: Educational Development Initiative.

Tasiu, M., Yusuf, H., & Bello, A. (2024). Dynamics of banditry: A mathematical modeling approach. *International Journal of Applied Mathematics and Statistics, 59*(1), 145–167.

UNICEF. (2022). *Education under attack in Nigeria: Impact of insecurity on schools and students*. Abuja: UNICEF Nigeria.

Usman, A. (2021). *Kidnapping and national security in Nigeria*. Lagos: Centre for Strategic Studies.

Wallace, C., & Silander, D. (2018). *Global security challenges in the 21st century*. New York, NY: Routledge.

Yakubu, M., & Abah, F. (2023). Mathematical modeling of banditry as an epidemic in Nigeria. *International Journal of Security and Mathematics, 7*(2), 99–115.