



Science Forum (Journal of Pure and Applied Sciences)



Journal Homepage: <https://atbuscienceforum.com.ng/index.php/jpas>

Analysis of the Operating Characteristic Curve in Acceptance Sampling Schemes for Quality Assurance

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Abstract

In modern manufacturing industries, acceptance sampling plans serve as indispensable tools for maintaining and verifying product quality. They are employed at various stages—covering raw materials, semi-finished products, and finished goods—to determine whether a production lot should be accepted or rejected based on the results of sample inspections. When a lot fails to meet quality specifications, it is either returned to the supplier or subjected to rectifying inspection, during which defective units are identified, replaced, or reworked. Although effective, this process often results in high inspection costs, longer production cycles, and increased operational inefficiencies. To address these challenges, this study proposes an economic model designed to optimize single and double sampling plans for improved efficiency and reliability in industrial quality assurance. The model aims to minimize the Average Total Inspection (ATI) while maintaining the desired levels of producer's and consumer's protection. This ensures that high-quality lots are not wrongly rejected and that substandard lots are not erroneously accepted. The developed approach provides a statistically grounded framework for balancing cost-effectiveness, risk minimization, and quality assurance within the production process. A comparative analysis of the optimized models for Single Sampling Plans (SSP) and Double Sampling Plans (DSP) demonstrates that the ATI generally increases with the fraction defective (p), with SSP yielding slightly higher ATI values compared to DSP. The results further revealed that the optimized DSP achieved a minimum consumer's risk of 0.0971, whereas SSP showed a relatively higher consumer's risk of 0.0979, indicating the superior performance and flexibility of the double sampling approach. Overall, the study highlights the critical role of optimal acceptance sampling design in improving inspection efficiency, reducing production costs, and enhancing decision-making across manufacturing and supply chain systems. The proposed economic model provides a practical and analytical framework for industries seeking to implement more effective and sustainable quality control strategies.

Keywords

OC Curve,
Acceptance sampling,
Quality assurance,
Sampling plans

ISSN 1119-4618



1119 4618

JPAS



— Faculty of Science, ATBU Bauchi —

Introduction

Quality control is a critical aspect of manufacturing and production processes for ensuring that products meet specified standards and customer expectations (Montgomery, 2013). One of the primary tools used in quality control is acceptance sampling, which involves inspecting a random sample of items from a lot to decide whether to accept or reject the entire lot. Acceptance sampling plans are characterized by their operating characteristics (OC) curves representing the probability of accepting a lot for given proportion of defective items. Basic factors in determining optimum sampling plans include sample size and acceptance number (Borrór and Montgomery, 2018) as they do influence the shape of OC curve.

The OC curve is a graphical representation of the likelihood that a lot will be accepted depending on its quality level. It is worthy of note that larger sample size typically provides more information about the lot, leading to a steeper OC curve that discriminates better between good and bad lots. Conversely, the acceptance number determines the threshold of defective items allowed in the sample before the lot is rejected. Adjusting these parameters can significantly impact the cost and effectiveness of quality control processes (Evans and Lindsay, 2014).

Single acceptance sampling plans involve inspecting one sample from a lot and making a decision based on the number of defective items found. Double acceptance sampling plans, on the other hand, involve two stages of sampling. If the number of defectives in the first sample is within an acceptable range, a second sample is taken to make the final

decision. This plan offers a better balance between inspection costs and quality assurance by providing a second chance to evaluate the lot before making a final decision (Borrór and Montgomery, 2018).

The impact of sample size on the OC curve in single and double acceptance sampling plans has been extensively studied. Increasing the sample size generally leads to a more accurate estimation of the lot quality, which improves the performance of the OC curve. However, larger sample sizes also increase inspection costs and time, presenting a trade-off that must be carefully managed. Researchers have developed various models and methods to optimize sample sizes, considering factors such as the cost of inspection, the cost of passing defective items, and the desired level of protection against accepting bad lots (Krishnamoorthi, 2016).

Similarly, the acceptance number is a critical parameter that affects the OC curve. A lower acceptance number means stricter acceptance criteria, which can reduce the risk of accepting defective lots but may also lead to higher rejection rates and associated costs. Conversely, a higher acceptance number makes the plan more lenient, potentially reducing inspection costs but increasing the risk of accepting poor-quality lots. The challenge lies in finding an optimal acceptance number that balances these competing factors. Studies have shown that both sample size and acceptance number must be carefully chosen to achieve the desired quality control objectives while minimizing costs (Montgomery, 2013).

The aim of this study is to examine an economic model that optimize single and double sampling plans for reducing inspection costs

and enhancing quality control in manufacturing. Specific objectives are to:

- i. evaluate the effects of Sample size (n) and Acceptance number (c) on the Operating Characteristics (OC) curves of Single and Double Sampling Plans;
- ii. determining and compare the optimal sampling plans and minimizing Average Total Inspection (ATI) for SSP and DSP sampling inspection;

METHODS

Probability of Accepting a Lot

The probability of acceptance is the probability of not detecting a defective unit in a sample taken from the lot. If x represents the true number of defective units in a sample n and with a binomial random variable ranging from 0 to n . The probability that x defective unit will be observed in a sample n is given as:

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x} \quad (1)$$

where p is the proportion of defective units.

Single Sampling Inspection Plan

In a single sampling inspection, a sample is randomly taken from a lot, N . If the number of defective units in a sample x is less than or equal to acceptance number c , the lot is accepted. However, if the number of defective units in the sample is more than the acceptance number, the lot is rejected. Probability of acceptance (P_a) for single sampling plan is given as:

$$P_a = p(x \leq c) = \sum_{x=0}^c \binom{n}{x} p^x (1 - p)^{n-x} \quad (2)$$

where p is the true fraction of defective units.

Average outgoing quality (AOQ) for single sampling plan (SSP)

If a lot is accepted, all observed defective units in the sample are replaced with units that are not defective. The remaining units that is not inspected is $(N - n)$. If fraction defective unit in the lot is p , then the fraction defective in the remaining units that are not inspected is $(N - n)p$. If the probability that the lot will be accepted is P_a , then the defective units in the accepted lot is given as $P_a p (N - n)$. However, if the lot is rejected, all defective units in the sample and in the rejected lot are replaced with non-defective units and the lot will contain no defective units. Thus, Average Outgoing Quality (AOQ) is given as:

$$AOQ_1 = \frac{(N - n)pP_a}{N}$$

Average total inspection for single sampling plan

Average total inspection (ATI) represents the average number of units inspected in the sample and in the rejected lot and is given as:

$$ATI = n + (1 - P_a)(N - n) \quad (4)$$

Double Sampling Plan (DSP)

The procedure for acceptance and reaction of lot in double sampling plan is as follows: A sample of size n_1 is drawn from the lot and inspected, if the defective units in the first sample x_1 is less than or equal to the first acceptance number c_1 the lot is accepted on the first sample. If the defective units in the first sample x_1 is greater than the second acceptance number c_2 the lot is rejected on the first sample. However, if x_1 greater than c_1 but less than or equal to c_2 , a second sample of

size n_2 is taken. If the combined defectives in the two samples $(x_1 + x_2)$ is less or equal to c_2 the lot is accepted or else the lot is rejected. The probability of acceptance on the first sample (P_{a1}) is:

$$\begin{aligned}
 P_{a1} &= P_r\{x_1 \leq c_1\} \\
 &= \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} \tag{5}
 \end{aligned}$$

The probability of acceptance on the second sample is:

$$\begin{aligned}
 P_{a2} &= \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} \right] \right. \\
 &\quad \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p^{x_2} (1-p)^{n_2-x_2} \right] \left. \right\} \tag{6}
 \end{aligned}$$

The probability of final acceptance of the lot is denoted by (P_a) and it is the sum of P_{a1} and P_{a2} . The acceptance probabilities depend on the fraction defective units p in the lot. Thus, the probability of final acceptance of the lot is:

$$\begin{aligned}
 P_a &= P_{a1} + P_{a2} = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} \\
 &\quad + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} \right] \right. \\
 &\quad \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p^{x_2} (1-p)^{n_2-x_2} \right] \left. \right\}
 \end{aligned}$$

Probability of rejection on the first sample is:

$$\begin{aligned}
 &1 - P_{a1} \\
 &= 1 - \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} \tag{8}
 \end{aligned}$$

Average outgoing quality (AOQ) for double sampling plan

Given that the lot of size N is accepted on the first sample of size n_1 , the remaining part of the lot $(N - n_1)$ with fraction defective p is not inspected. The defective units in the portion of the lot that is not inspected is $(N - n_1)p$. The expected number of defective units in the accepted lot is $p(N - n_1)P_{a1}$.

If the lot is rejected on the first sample, it means the rejected lot will be subjected to 100% inspection with all the units that are defective replaced with the units that are not defective. The probability of rejection of the lot on the first sample is $(1 - P_{a1})$. Therefore the expected number of defective units per outgoing lot is zero.

Assuming the lot is accepted on the second sample, then the portion of the lot $(N - n_1 - n_2)$ is not inspected. Given that p is the fraction defective units it shows that the proportion of defective units in the accepted lot is $(N - n_1 - n_2)p$. Given also that the probability of acceptance of the lot on the second sample is P_{a2} , then the proportion of defective units in the outgoing lot which is accepted is $p(N - n_1 - n_2)P_{a2}$.

Assuming there is rejection of the lot on the second sample with probability $(1 - P_{a2})$, it then follows that the rejected lot is to be subjected to 100% inspection with all the units that are defective are replaced with the units that are not defective and the outgoing lot has zero defectives.

Therefore, there are defective units in the accepted lot based on the first sample as well as in the accepted lot based on the second sample. The combined expected number of

defective units is $p(N - n_1)P_{a1} + p(N - n_1 - n_2)P_{a2}$

The Average Outgoing Quality (AOQ) for rectifying double sampling plan is given as:

$$AOQ_1 = \frac{p[P_{a1}(N - n_1) + P_{a2}(N - n_1 - n_2)]}{N}$$

Average total inspection (ATI) for double sampling plan

In double sampling plan, the probability that the lot is accepted on the first sample n_1 is only P_{a1} . Given that it is accepted on the second sample it means that two samples ($n_1 + n_2$) are inspected with the probability of acceptance P_{a2} and if the whole lot N is rejected, with the probability $(1 - P_{a1} - P_{a2})$, the average total inspection is calculated as:

$$\begin{aligned} ATI &= n_1P_{a1} + (n_1 + n_2)P_{a2} \\ &+ N(1 - P_{a1} - P_{a2}) \end{aligned} \quad (10)$$

Design of Sampling Plans with Specified AQL and LTPD

Two points of interest namely $(p_1, 1 - \alpha)$ and (p_2, β) are specified on the operating characteristics (OC) curve in the design of acceptance sampling plan. P_1 is Acceptable Quality Level (AQL), P_2 is Lot Tolerant Percent Defective (LTPD), α is the producer's risk and β is the consumer risk. Acceptable Quality Level (AQL) is the small percentage of defective units in a lot that is acceptable by the consumer. Rejecting a lot with acceptable quality level results to producer risk (α). Also Lot Tolerant Percent Defective (LTPD) is the highest percentage of defective units a consumer can tolerate in a lot. Accepting lot with this quality level results to consumer's risk (β). Acceptable Quality Level (AQL) of 0.02 with a producer's risk (α) = 0.05 is used in the design of Single and Double sampling

plans such that the probability of rejection of the lot given acceptable quality level (AQL) should be less than or equal to the value of the producer's risk (α). On the other hand, Lot Tolerant Percent Defective (LTPD) of 0.07 with consumer's risk (β) = 0.1 is also used in this design such that the probability of acceptance of the lot given lot Tolerant Proportion Defective (LTPD) is less than or equal to the value of the consumer's risk (β). Probability of acceptance in single sampling plan given AQL and LTPD levels of quality

In the design of acceptance single sampling plan, we chose an appropriate sample size (n) and acceptance number (c). Given the probabilities of acceptance (P_a) with associated quality level $p_1 = AQL$, and $p_2 = LTPD$. Probability of lot acceptance is thus given as:

$$\begin{aligned} 1 - \alpha &= P_a(x \leq c | n, p_1 = AQL) \\ &= \sum_{x=0}^c \binom{n}{x} AQL^x (1 - AQL)^{n-x} \end{aligned} \quad (11)$$

$$\begin{aligned} 1 - \alpha &= P_a(AQL) \end{aligned}$$

The probability of rejection at $p_1 = AQL$ or producer's risk is:

$$\begin{aligned} 1 - P_a(AQL) &= \alpha \end{aligned} \quad (13)$$

On the other hand, if the lot with quality level $p_2 = LTPD$ is accepted, then there is consumer's risk. Thus, the probability of accepting lot with quality level $p_2 = LTPD$ is stated below:

$$\begin{aligned} \beta &= P(x \leq c | n, p_2 = LTPD) \\ &= \sum_{x=0}^c \binom{n}{x} p_2^x (1 - p_2)^{n-x} \\ \beta &= \sum_{x=0}^c \binom{n}{x} LTPD^x (1 - LTPD)^{n-x} \\ \beta &= P_a(LTPD) \end{aligned}$$

Probability of acceptance for double sampling plan given AQL and LTPD levels of quality.

In the design of Double Sampling Plan, an appropriate sampling plan n_1, n_2, c_1 and c_2 is found and the probabilities of acceptance at a quality level $p_1 = AQL$ and $p_2 = LTPD$ is formulated. Thus, the probabilities of lot acceptance given quality level $p_1 = AQL$ and $p_2 = LTPD$ are determined as shown below:

$$1 - \alpha = P_{a_1} + P_{a_2} = P(x_1 \leq c_1 | n_1, p_1 = AQL) + P(x_1 + x_2 \leq c_2 | c_1 < x_1 \leq c_2, n_1, n_2, p_1 = AQL) = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} AQL^{x_1} (1 - AQL)^{n_1 - x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} AQL^{x_1} (1 - AQL)^{n_1 - x_1} \right] \times \left[\sum_{x_2=0}^{c_2 - x_1} \binom{n_2}{x_2} AQL^{x_2} (1 - AQL)^{n_2 - x_2} \right] \right\}$$

$$1 - \alpha = P_a(AQL)$$

Average Total Inspection (ATI) Minimization Model

Suppose set lots are to be inspected in order to take a decision to either accept or reject the lots. The main goal of the inspector is to minimize the average total inspection (ATI) of the lots by choosing an optimal sample size so that both the producer's risk and the consumer's risk are reduced. A model to achieve optimality in the Average Total Inspection (ATI) with a small sample size such that both the producer's and the consumer's risks are minimized is as given below:

Minimize: (ATI)
 Maximize: $P_a(p)$
 Subject to $1 - P_a(AQL) \leq \alpha$
 $P_a(LTPD) \leq \beta$

Producer's risk is then given as: $1 - P_a(AQL) = \alpha$ (17)

$$\beta = P_{a_1} + P_{a_2} = P(x_1 \leq c_1 | n_1, p_2 = LTPD) + P(x_1 + x_2 \leq c_2 | c_1 < x_1 \leq c_2, n_1, n_2, p_2 = LTPD)$$

(18)

$$\beta = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} LTPD^{x_1} (1 - LTPD)^{n_1 - x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} LTPD^{x_1} (1 - LTPD)^{n_1 - x_1} \right] \times \left[\sum_{x_2=0}^{c_2 - x_1} \binom{n_2}{x_2} LTPD^{x_2} (1 - LTPD)^{n_2 - x_2} \right] \right\}$$

(15)

$$\beta = P_a(LTPD)$$

(16)

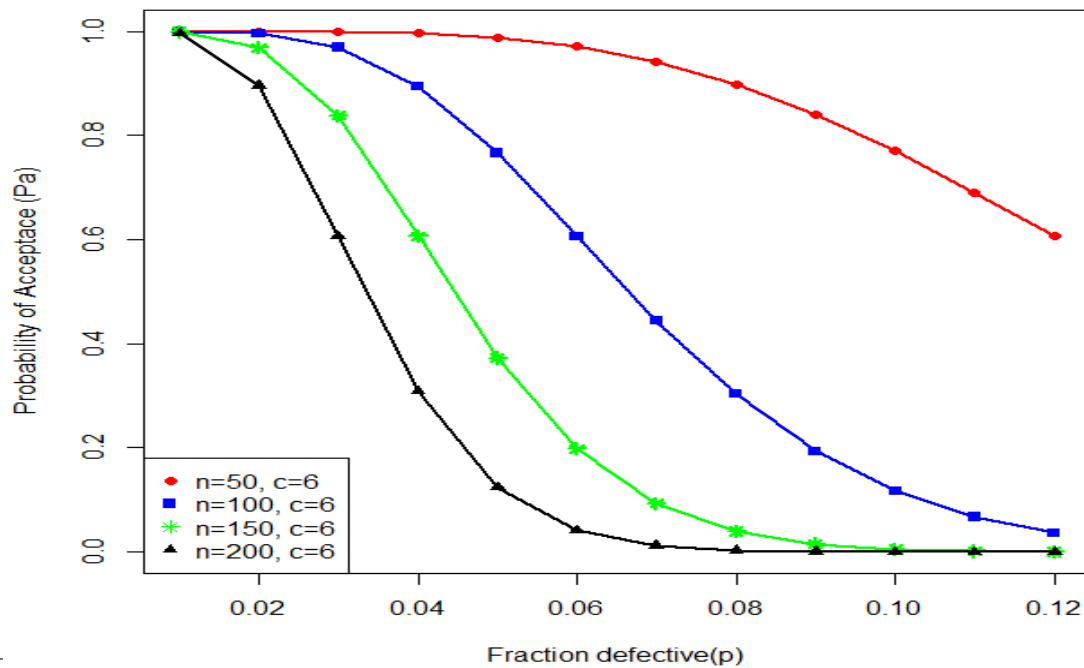
$1 - P_a(AQL)$ is the probability of rejection of the lot at acceptable quality level, $P_a(LTPD)$ is the probability of acceptance of the lot given Lot tolerance percent defective. In addition, α and β are the values of producer's risk and consumer's risk respectively.

RESULTS

The effect of sample size and acceptance number on the Operating Characteristics (OC) curve of Single Sampling Plan (SSP) and Double Sampling Plan (DSP) is investigated and the results are presented on Tables 1-7 and Figures 1-5.

Table 1: Operating Characteristics (OC) Curve for SSP with increasing sample size (n)

p	$P_a(p), n = 50, c = 6$	$P_a(p), n = 100, c = 6$	$P_a(p), n = 150, c = 6$	$P_a(p), n = 200, c = 6$
0.01	0.9999	0.9999	0.9992	0.9957
0.02(AQL)	0.9999	0.9959	0.9680	0.8957
0.03	0.9992	0.9687	0.8340	0.6063
0.04	0.9992	0.8936	0.6063	0.3083
0.05	0.9882	0.7660	0.3729	0.1237
0.06	0.9711	0.6063	0.1984	0.0413
0.07(LTPD)	0.9417	0.4442	0.0934	0.0119
0.08	0.8981	0.3031	0.0397	0.0030
0.09	0.8404	0.1939	0.0155	0.0007
0.10	0.7702	0.1172	0.0056	0.0001
0.11	0.6909	0.0672	0.0019	0.0000
0.12	0.6065	0.0367	0.0006	0.0000

Fig. 1: Plot of sample size (n) on the OC of SSP

The effect of increase in sample size on the operating characteristics (OC) curve of Single Sampling Plan (SSP) is shown in Table 1 and Figure 1. Increasing the sample size and keeping the acceptance number constant makes it difficult for the lot to be accepted by the consumer. This is because the chances of finding more defective units is greater when the sample size is large.

Table 1 Figure 1: specifically as fraction of defective is 0.02, probability of acceptance is approximately 90% for $n = 200$, 98% for $n = 150$, 100% for $n = 100$ and $n = 50$. Similarly, as proportion defective increases to say 6% (0.06), probability of acceptance is less than 10% for $n = 200$, 20% for $n = 150$ and 60% for $n = 100$ and about 98% for $n = 50$. Thus, increasing fraction defective for given sample size decreases probability of acceptance for fraction defective $p > 0.02$ (or increase

probability of rejection). Consequently, maintaining $n = 50$ provides more economical sample for higher probability of acceptance (lower probability of rejection) in a single sampling plan.

This tends to increase the consumers risk at higher levels of fraction defective p . This probability of acceptance is however reduced for higher sample size. At $n = 200$, the probability of rejection tends to 10% for 6% proportion defective: P while higher sample size protects the consumer by minimizing fraction defective. It is noticeable that for 50% probability of acceptance about 3.5%, 4.5% 6.5% more than 12% proportion defectives will be accommodated. This strikes a balance for both consumers and producers in a single sampling plan.

Table2: Operating Characteristics (OC) Curve for SSP with increasing acceptance number (c)

p	$P_a(p); n = 50, c = 6$	$P_a(p); n = 50, c = 7$	$P_a(p); n = 50, c = 8$	$P_a(p); n = 50, c = 9$
0.01	0.9999	0.8999	0.9999	1.0000
0.02(AQL)	0.9999	0.9999	0.9989	0.9999
0.03	0.9992	0.9998	0.9899	0.9999
0.04	0.9963	0.9992	0.9898	0.9999
0.05	0.9882	0.9968	0.9992	0.9998
0.06	0.9711	0.9906	0.9973	0.9993
0.07(LTPD)	0.9417	0.9779	0.9927	0.9978
0.08	0.8981	0.9562	0.9833	0.9944
0.09	0.8404	0.9232	0.9672	0.9875
0.10	0.7702	0.8779	0.9421	0.9755
0.11	0.6909	0.8207	0.9068	0.9565
0.12	0.6065	0.7533	0.8607	0.9292

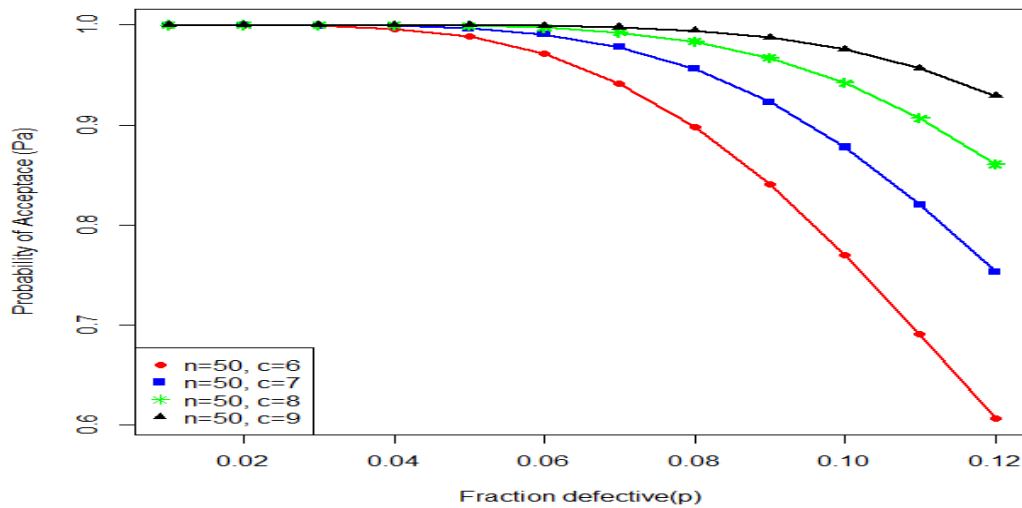


Fig. 2: Plot of increase in Acceptance number (c) on the OC curve of SSP

On Table 2 and Figure 2, when sample size is fixed at 50 while allowing number of defects to vary from 6 to 9, it is noticeable that as fraction defectives p increase probability of acceptance decreases. Specifically, for $P < 0.04$ the probability of acceptance is about 100% in all cases but reduces to about 0.62 for $p = 0.12$. therefore, fixing sample size (n) and increase acceptance number (c) will obviously increase the probability of acceptance for

higher p and decrease probability for acceptance for lower p . In other words, it increases probability of rejection for higher p and decrease probability of rejection for lower p . Thus, for sample of size $n = 50$ and $c = 6, 7, 8, 9$ accommodating p in the range of 0.01 to 0.06 will provide higher probability of acceptance. Consequently, increasing c will not be economical for a single sampling plan with $n=50$.

Table3: Operating Characteristics (OC) Curve for DSP with increasing sample size (n)

p	$P_a(p) n1=66, n2=132, c1=1, c2=1$	$P_a(p) n1=68, n2=136, c1=1, c2=8$	$P_a(p) n1=70, n2=140, c1=1, c2=8$	$P_a(p) n1=72, n2=144, c1=1, c2=8$
0.01	0.9998	0.9998	0.9997	0.9997
0.02(AQL)	0.9832	0.9801	0.9765	0.9726
0.03	0.8717	0.8546	0.8365	0.8173
0.04	0.6394	0.6074	0.5755	0.5437
0.05	0.3859	0.3526	0.3211	0.2916
0.06	0.9999	0.1756	0.1538	0.1345
0.07(LTPD)	0.0941	0.0799	0.0678	0.0576
0.08	0.0426	0.0353	0.0294	0.0245
0.09	0.0194	0.0159	0.0131	0.0108
0.10	0.0092	0.0074	0.0060	0.0049
0.11	0.0045	0.0036	0.0029	0.0023
0.12	0.0022	0.0018	0.0014	0.0011

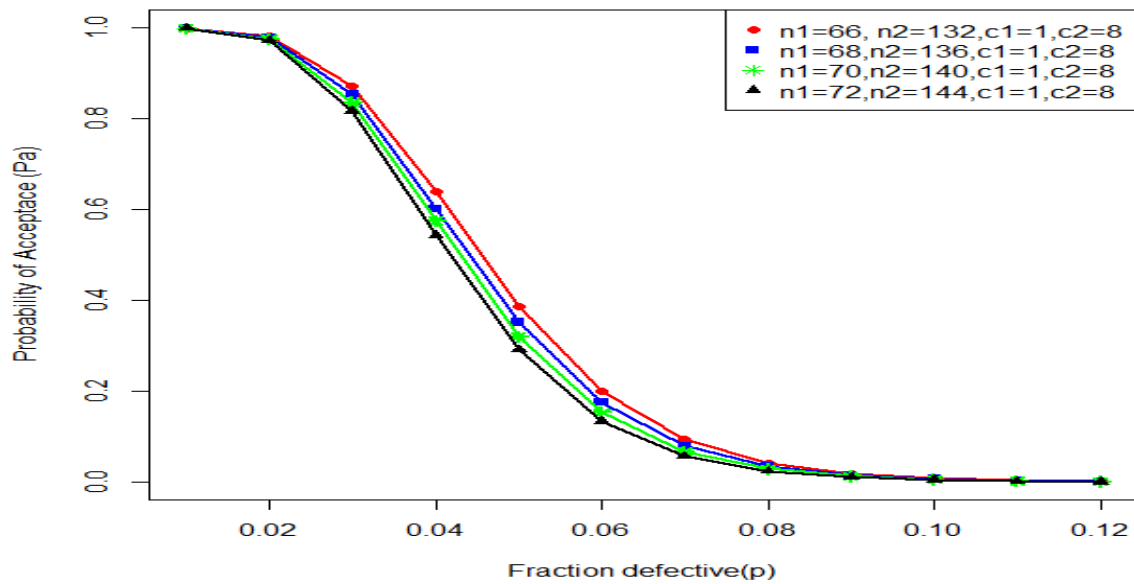


Fig. 3: Plot of increasing Sample size on the OC curve of DSP

On Table 3 and Figure 3, as the proportion defectives increase probability of acceptance decreases. In all the sample plans, 2% fraction defective show a nearly 100% probability of acceptance (or a near zero probability of rejecting the lots). The sample with $n_1=72, n_2 =$

144 for $c_1, c_2 = 1$ and 8 provides slightly lower probability of acceptance at higher level of fraction defective while the sample with $n_1 = 66, n_2 = 132$, then $c_1 = 1, c_2 = 8$ provided higher probability of acceptance from fraction defective of between 0.03 to 0.06.

Table 4: Operating Characteristics (OC) Curve for DSP with increasing acceptance number (c)

p	$P_a(p)_{n1=66, n2=132, c1=1, c2=8}$	$P_a(p)_{n1=66, n2=132, c1=2, c2=10}$	$P_a(p)_{n1=66, n2=132, c1=3, c2=11}$	$P_a(p)_{n1=66, n2=132, c1=4, c2=12}$
0.01	0.9999	1.0000	1.0000	1.0000
0.02(AQL)	0.9873	0.9987	0.9795	0.9999
0.03	0.9001	0.9778	0.9887	0.9964
0.04	0.7092	0.8934	0.9306	0.9708
0.05	0.4848	0.7334	0.7945	0.8979
0.06	0.3006	0.5421	0.6050	0.7569
0.07(LTPD)	0.1773	0.3709	0.4173	0.5919
0.08	0.1028	0.2422	0.2689	0.4343
0.09	0.0593	0.1542	0.1668	0.3052
0.10	0.0340	0.0966	0.1017	0.0284
0.11	0.0193	0.0597	0.0614	0.1393
0.12	0.0109	0.0362	0.0368	0.0912

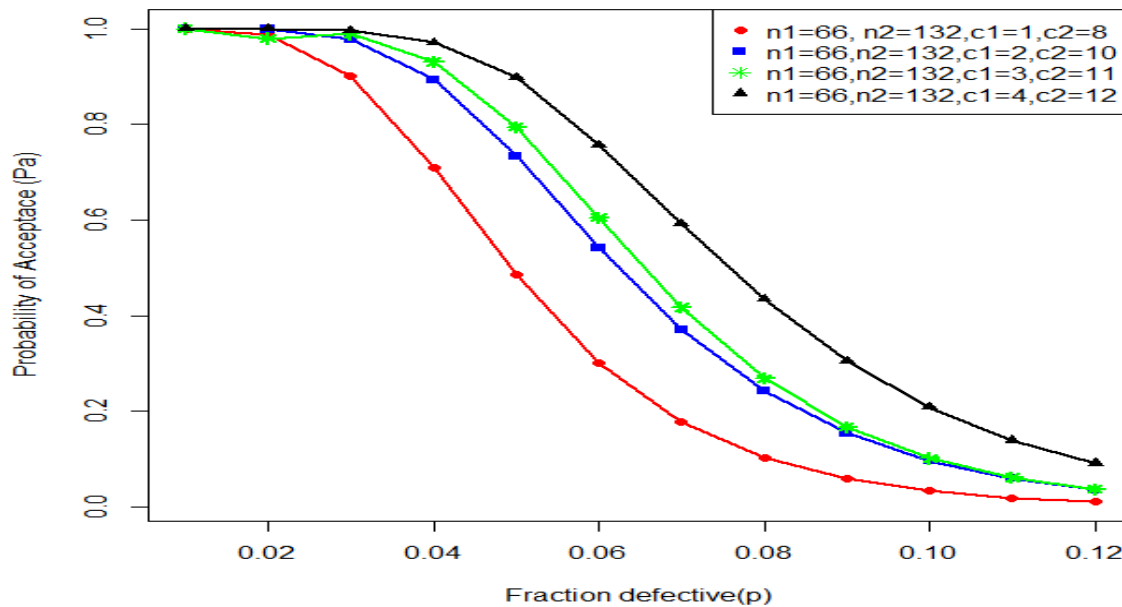


Fig. 4: Plot of Acceptance number (C) on the OC curve of DSP

Table 4 Figure 4: The operating characteristic curve (OC) expands from the left to right as n_1 and n_2 are fixed while c_1 and c_2 increases. This shows nearly equal probability of acceptance for proportion defective between 10% to 12% but increasing probability for proportion defective between 2% and 8% converging to 100% for 2% proportion defective.

Determination of optimal sampling plans for Single Sampling and Double Sampling Plans
Optimal sampling plans for Single Sampling Plan (SSP) and Double Sampling Plan (DSP) would provide the desired sample size, minimize the Average Total Inspection (ATI) and also keep both $1 - P_a(AQL)$ and $P_a(LTPD)$ within the required threshold of ≤ 0.05 and ≤ 0.10 respectively are computed and reported in Tables 5 and 6.

Table 5: Single Sampling Plans (SSP) with the parameters ($N = 1000, AQL = 0.02, LTPD = 0.07, \alpha = 0.05, \beta = 0.1, AOQL = 0.03$ with $n \leq 205$)

n	c	AOQ	ATI	$1 - P_a(AQL)$	$P_a(LTPD)$	$P_a(p)$
196	7	0.0184	386.83	0.0448	0.0322	0.7627
196	8	0.0208	306.47	0.0180	0.0642	0.8626
197	7	0.0183	390.88	0.0459	0.0309	0.7586
197	8	0.0207	309.74	0.0185	0.0620	0.8596
198	7	0.0182	394.95	0.0470	0.0297	0.7544
198	8	0.0206	313.04	0.0190	0.0598	0.8566
199	7	0.0180	399.03	0.0482	0.0285	0.7503
199	8	0.0205	316.35	0.0196	0.0577	0.8535

200	7	0.0179	403.12	0.0493	0.0274	0.7461
200	8	0.0204	319.68	0.0202	0.0556	0.8504
201	8	0.0203	323.03	0.0208	0.0537	0.8473
201	9	0.0220	267.19	0.0077	0.0979	0.9172
202	8	0.0202	326.40	0.0214	0.0518	0.8441
202	9	0.0219	269.78	0.0080	0.0947	0.9151
203	8	0.0201	329.79	0.0220	0.0499	0.8409
203	9	0.0218	272.39	0.0083	0.0917	0.9129
204	8	0.0200	333.19	0.0226	0.0481	0.8377
204	9	0.0217	275.03	0.0085	0.0888	0.9108
205	8	0.0199	336.62	0.0232	0.0464	0.8344
205	9	0.0217	277.68	0.0088	0.0859	0.9086

Table 5 shows different sampling plans generated for Single Sampling Plan (SSP) with Average Total Inspection (ATI), probability of rejection of the lot given acceptable quality level $1 - P_a(AQL)$, probability of acceptance given Lot Tolerant Percent defective units $P_a(LTPD)$ and probability of acceptance of the lot $P_a(p)$ for all the sampling plans. However, the optimal sampling plan is obtained at $n = 201, c = 9$ with a minimum Average Total Inspection (ATI) of 267.19 and higher probability of acceptance $P_a(p)$ of 0.9172 satisfying both the producer's risk of $1 - P_a(AQL) \leq 0.05$ and consumer's risk of $P_a(LTPD) \leq 0.1$ respectively.

Table 6 shows different sampling plans generated for Double Sampling Plan (DSP) with Average Total Inspection (ATI), combined probability of rejection of the lot given acceptable quality level $1 - P_a(AQL)$, combined probability of acceptance given Lot Tolerant Percent defective units $P_a(LTPD)$ and combined probability of acceptance of the lot $P_a(p)$ for all the sampling plans. However, the optimal sampling plan is obtained at $n_1 = 96, n_2 = 192, c_1 = 3, c_2 = 11$, with a minimum Average Total Inspection (ATI) of 237.32 and higher probability of acceptance $P_a(p)$ of 0.8891 satisfying both the producer's of $1 - P_a(AQL) \leq 0.05$ and consumer's risk of $P_a(LTPD) \leq 0.1$ respectively.

Table 6: Double Sampling Plan (DSP) Plans with the given parameters ($N = 1000, AQL = 0.02, LTPD = 0.07, \alpha = 0.05, \beta = 0.1, AOQL = 0.03$, with n_1 and $n_2 \leq 205$)

n_1	n_2	c_1	c_2	AOQ	ATI	$1 - P_a(AQL)$	$P_a(LTPD)$	$P_a(p)$
66	132	1	8	0.0226	247.16	0.0168	0.0941	0.8717
67	134	1	8	0.0223	256.78	0.0183	0.0867	0.8632
68	136	1	8	0.022	266.57	0.0199	0.0799	0.8546
69	138	1	8	0.0217	276.53	0.0216	0.0736	0.8456
70	140	1	8	0.0214	286.66	0.0235	0.0678	0.8365
71	142	1	8	0.0211	296.94	0.0254	0.0625	0.8270
72	144	1	8	0.0208	307.35	0.0274	0.0576	0.8173
73	146	1	8	0.0205	317.90	0.0295	0.0530	0.8075
74	148	1	8	0.0201	328.57	0.0318	0.0489	0.7973

75	150	1	8	0.0198	339.35	0.0342	0.0450	0.7870
76	152	1	8	0.0195	350.23	0.0367	0.0415	0.7465
77	154	1	8	0.0192	361.19	0.0393	0.0383	0.7658
77	154	2	8	0.0215	282.40	0.0300	0.0953	0.8146
78	156	1	8	0.0188	372.22	0.0420	0.0353	0.7550
78	156	2	8	0.0212	291.74	0.0321	0.0901	0.8057
79	158	1	8	0.0185	383.32	0.0448	0.0326	0.7439
79	158	2	8	0.0210	301.17	0.0343	0.0852	0.7966
80	160	1	8	0.0182	394.48	0.0478	0.0301	0.7328
80	160	2	8	0.0207	310.68	0.0366	0.0805	0.7874
81	162	2	8	0.0204	320.27	0.0390	0.0761	0.7780
82	164	2	8	0.0201	329.92	0.0415	0.0719	0.7686
83	166	2	8	0.0198	339.63	0.0441	0.0680	0.759
84	168	2	8	0.0195	349.39	0.0469	0.0643	0.7493
85	170	2	8	0.0192	359.18	0.0497	0.0608	0.7395
95	190	3	10	0.0221	263.69	0.0175	0.0972	0.8488
96	192	3	10	0.0219	271.07	0.0187	0.0926	0.8419
96	192	3	11	0.0229	237.32	0.0093	0.0971	0.8891
97	194	3	10	0.0216	278.10	0.0200	0.0882	0.8349
97	194	3	11	0.0227	244.10	0.0100	0.0923	0.8835
98	196	3	10	0.0214	286.10	0.0213	0.0841	0.8277
98	196	3	11	0.0225	250.98	0.0108	0.0878	0.8775

Table 7: Effect of fraction defective (p) on the Average Total Inspection (ATI) in optimal Single Sampling Plan (SSP) and Double Sampling Plan (DSP)

Optimal Single Sampling Plan (SSP) $n=201, c=9$		Optimal Double Sampling plan (DSP) $n_1 = 96, c_1=3, n_2 = 192, c_2 = 11$	
p	ATI	p	ATI
0.01	201.03	0.01	99.10
0.02	207.18	0.02	126.99
0.03	267.19	0.03	237.32
0.04	429.41	0.04	454.21
0.05	641.79	0.05	676.94
0.06	816.99	0.06	829.04
0.07	921.86	0.07	913.65
0.08	971.36	0.08	957.32
0.09	990.79	0.09	979.38
0.10	997.35	0.10	990.31
0.11	999.30	0.11	999.59
0.12	999.80	0.12	998.05

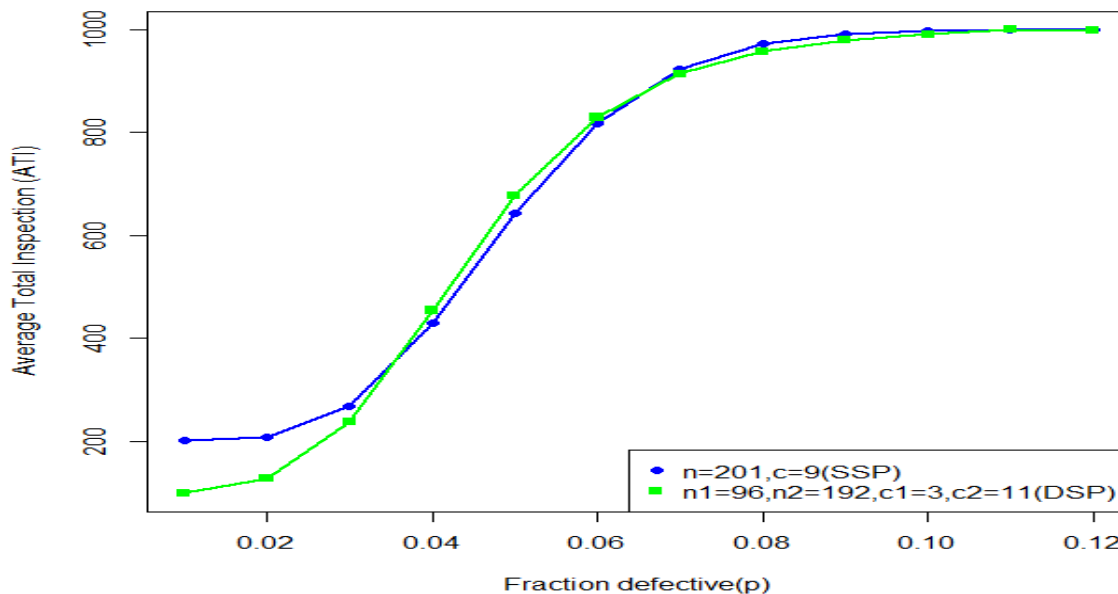


Fig.5: Average Total Inspection ATI of optimal SSP and DSP

In Figure 5, the effect of fraction defective units of the Average Total Inspection (ATI) of the optimal sampling plans in both Single and Double sampling plans is compared. The Average Total Inspection (ATI) for both plans are seen to have increased and converged as the fraction defective units increased. However, the Average Total Inspection (ATI) for optimal Single sampling plan is generally higher than that of the optimal Double sampling plan due to the larger sample size of the optimal Single sampling plan.

Conclusion

This study demonstrated that in a Single Sampling Plan (SSP), an increase in the fraction defective (ppp) for a fixed sample size (nnn) leads to a corresponding decrease in the probability of acceptance, particularly when $p > 0.02$. This implies that as product quality deteriorates, the likelihood of accepting a defective lot significantly reduces—ensuring consumer protection. However, maintaining a sample size of $n=50$ =

50n=50 was found to provide an optimal balance between inspection economy and decision reliability. At this sample size, the optimal Average Total Inspection (ATI) of 267.19 was achieved, satisfying both the producer's and consumer's risks, and minimizing unnecessary inspection costs.

For the Double Sampling Plan (DSP), the optimized parameters were $n_1=96, n_2=192, c_1=3, c_2=11$. These values produced an optimal ATI of 237.32, indicating a more efficient inspection strategy compared to SSP. The DSP demonstrated higher flexibility and cost-effectiveness, as it allowed for early lot acceptance or rejection after the first sample, reducing the overall inspection effort. Overall, the study highlights that while both SSP and DSP provide reliable quality control mechanisms, the double sampling plan offers superior performance in terms of minimizing inspection costs and maintaining satisfactory producer and consumer protection levels. The developed economic model thus provides a

valuable decision-support tool for optimizing sampling procedures in manufacturing industries, contributing to improved product quality, reduced wastage, and enhanced operational efficiency.

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