https://doi.org/ 10.70882/7cqv7d58

SCIENCE FORUM (JOURNAL OF PURE AND APPLIED SCIENCES) 26, (4) (2026), 419 - 425

Science Forum (Journal of Pure and Applied Sciences)

Journal Homepage: https://atbuscienceforum.com.ng/index.php/jpas

Ideal Mappings between Ideal Spaces: Contrast, Exhaustiveness, Correspondence, and Homeomorphism

Ahmed Lafta Mosa Alhindawe [1], **Maryam Ameen Oleiwi** [2]

[1]*Department of Mathematics, Faculty of Computer Sciences and Mathematics, University of Kufa, Najaf, Iraq*

[2]*Department of Aeronautical Technical Engineering, Technical Engineering College of Najaf, Al-Furat Al-Awsat Technical University, Najaf, Iraq*

Abstract

Ideals of subsets can be used in a theory of ideal topological spaces, which generalizes classical topological concepts. Several notions of continuity and related mappings have been studied in ideal spaces, but there is the need for a unified definition of mappings by ideal sets, which is not using the notion of continuity. In this paper, several kinds of mappings between ideal spaces are introduced and investigated: ideal contrast mappings, ideal exhaustive mappings, and ideal correspondence mappings. Further, the notion of mappings which are ideal continuous is also taken into consideration, and ideal homeomorphisms are described with the help of the introduced mapping properties. The basic goal is to build basic relationships between these mappings and to determine sufficient conditions for the preservation of their properties. Various propositions and theorems are set up such as characterizing ideal homeomorphisms with bijective-ideal correspondence mappings. The results show that the concepts introduced here offer a systematic way of studying mappings between ideal spaces, while only the ideal structures related to the sets on which they operate are needed. The results are used to advance the theory of ideal topological spaces, and serve as a basis for further studies of generalized mappings and structural relationships between ideal spaces.

Keywords

Ideal spaces; Ideal mappings; Ideal contrast; Ideal exhaustive; Ideal correspondence; Ideal continuous mapping; Ideal homeomorphism.

EISSN: 3156-5190 (Online)
ISSN: 1119-4618 (Print)
JPAS
Faculty of Science, ATBU Bauchi

Corresponding Author: Ahmed Lafta Mosa Alhindawe, Department of Mathematics, Faculty of Computer Sciences and Mathematics, University of Kufa, Najaf, Iraq. Email: ahmedl.alhindawe@uokufa.edu.iq

1. Introduction

The theory of ideal topological spaces gives an important framework in which to extend classical topological structures considering distinguished families of subsets, which are hereditary and finite additive. Kuratowski developed this concept of an ideal, and Vaidyanathaswamy the localization theory relating to ideals. Later, Jankovic and Hamlett introduced significant ideal induced topologies and Hayashi and others studied the local properties and compactness in the ideal topological spaces [1-6]. These basic developments served as a foundation for the study of generalized openness, continuity properties, separation properties, compactness and related operators in ideal topological spaces.

Research on ideal topological spaces has been continued in recent years to the generalized open sets, the properties of continuity and new set-theoretic operators. Boonpok and Sama-Ae studied submaximality, connectedness and δβI-continuous mappings in ideal spaces which shows the ongoing importance of generalized continuity in this field [7]. Almuhur, et al. investigated hα-almost-weakly continuous functions and other properties of hα-open sets, one of the properties of generalized openness and continuity in ideal spaces [8,9]. Kong-ied et al. gave a unified theory of near continuity for mappings between ideal topological spaces and bitopological spaces [10]. Güldürdek and Açıkgöz have studied idealisation of h-open sets from the point of view of interior operators and continuity properties as well [11]. Other recent studies have brought about new set-operators, hΓ-open sets, and new dimensions connected to ideal topological spaces [12-14].

Ideal topology is one of the important directions of the study of mappings and continuity. In recent investigations [15–19] fuzzy ideal continuous mappings and applications of fuzzy open sets related to ideals in ideal topological spaces and various classes of continuous functions in ideal topological spaces have been studied. Njamcul and Pavlović included several types of continuity and their corresponding interpretation in ideal topological spaces [17] and Abdulraouf and Abd Al-Zhara looked at continuity of functions in ideal topological spaces [20] with several types of continuous functions. The results of these studies suggest that generalized mapping and continuity structures are still a field of study.

Although these advances have been made, the relationships of mappings that are explicitly given by the ideal subsets of the domain and codomain have not been sufficiently unified. Specifically, it is still needed to have a systematic description of complementary directional relationships between ideal subsets without using any other structures than the related ideals. This gives the impetus for the present study. For this reason, this paper proposes and explores three kinds of mappings between ideal spaces, namely ideal contrast mappings, ideal exhaustive mappings and ideal correspondence mappings.

An ideal contrast mapping is an ideal relation from ideal subsets of the codomain to ideal subsets of the domain and an ideal exhaustive mapping is an ideal relation from ideal subsets of the domain to ideal subsets of the codomain. An ideal correspondence mapping is a combination of these two opposing properties. The relationships between their relationships with ideal continuous mappings and with ideal homeomorphisms are then explored.

It is the tasks to formulate the propositions and theorems that describe the relationships between these concepts of mapping, such as between the bijective ideal correspondence mappings and the ideal homeomorphisms, which are the main contribution of this work. The properties of the introduced properties are also explored under injective, surjective and bijective maps. Therefore, the study gives a brief description of the formal structure for mapping between ideal spaces by direct mapping between the ideal subsets of these spaces.

2. Preliminaries

Definition 2.1 [1]: A collection subset $\mathfrak{I}$ of a given set $\mathcal{X}$ is rumored that an *ideal* on $\mathcal{X}$ if these conditions are holds:

1. Hereditary property: If A is in $\mathfrak{I}$ and $B \sqsubseteq A$, implies B is in $\mathfrak{I}$.
2. Finitely additive property: If A and $\mathcal{B}$ are in $\mathfrak{I}$ implies $A \cup \mathcal{B}$ in $\mathfrak{I}$.

Any subset of $\mathfrak{I}$ is said to be an $\mathfrak{I}$-*set*, also $(\mathcal{X}, \mathfrak{I})$ is said to be an *ideal space*. An ideal $\mathfrak{I}$ is described as *proper* if $\mathcal{X} \notin \mathfrak{I}$.

Example 2.2 [3]: Each of the following are examples of ideal:

1. $\mathfrak{I} = \{\phi\}$
2. $\mathfrak{I} = P(\mathcal{X})$.
3. $\mathfrak{I}_{\boldsymbol{\mathcal{F}in}}$ (all subsets are finite sets in $\mathcal{X}$).
4. $\mathfrak{I}_{\boldsymbol{c}}$ (all subsets are countable sets in $\mathcal{X}$).
5. $\mathfrak{I}_{\boldsymbol{n}}$ (all subsets are nowhere dense sets in $\mathcal{X}$).

Proposition 2.3 [6]: Let $\mathcal{F}: \mathcal{X} \to \Upsilon$ be an injective map and $\mathfrak{I}$ be an ideal defined on $\mathcal{X}$. Implies $\mathcal{F}(\mathfrak{I}) = \{\mathcal{F}(\mathcal{A}) : \mathcal{A} \in \mathfrak{I}\}$ is an ideal on Υ.

Proposition 2.4 [6]: - Let $\mathcal{F}: \mathcal{X} \to \Upsilon$ be an onto mapping and $\mathcal{T}$ be an ideal defined on Y, then, $\mathcal{F}^{-1}(\mathcal{T}) = \{\mathcal{F}^{-1}(\mathcal{B}) : \mathcal{B} \in \mathcal{T}\}$ is an ideal on $\mathcal{X}$.

3. Results

Definition 3.1: For two spaces X and Y, a mapping $\mathcal{F}: (X, \mathfrak{I}) \to (\Upsilon, \mathcal{T})$ is named *ideal contrast* if, for each $\mathfrak{I}$-set $\mathcal{A}$ of $\mathcal{X}$, we can find $\mathcal{T}$- set B in Υ such that $\mathcal{F}(\mathcal{A}) = \mathcal{B}$.

Example 3.2: Let $\mathcal{X} = \{1, 2, 3\}$, $I = \{\emptyset, \{1\}\}$, $\Upsilon = \{a, \mathfrak{b}, \mathfrak{c}\}$ and $\mathcal{T} = \{\emptyset, \{\mathfrak{b}\}, \{\mathfrak{c}\}, \{\mathfrak{b}, \mathfrak{c}\}\}$. If $\mathcal{F}: (\mathcal{X}, \mathfrak{I}) \to (\Upsilon, \mathcal{T})$ defined as : $\mathcal{F}(1) = b$, $\mathcal{F}(2) = a$ and $\mathcal{F}(3) = a$. Clearly, $\mathcal{F}$ is ideal contrast.

Definition 3.3: let $\mathcal{F}: (\mathbb{X}, \mathfrak{I}_{\mathbb{X}}) \to (\mathbb{Y}, \mathcal{T}_{\mathbb{Y}})$ be any mapping. $\mathcal{F}$ is an *ideal* exhaustive if for each $\mathcal{T}_{\mathbb{Y}}$-set $\mathcal{B}$, there exists an $\mathfrak{I}_{\mathbb{X}}$ -set $\mathcal{A}$, such that $\mathcal{F}(\mathcal{A}) \subseteq \mathcal{B}$.

Note that, in the above definition, if $\mathcal{F}$ is onto then $\mathcal{F}(\mathcal{A}) = \mathcal{B}$.

Example 3.4: Consider example 3.2, $\mathcal{F}$ is not ideal exhaustive. Since $\{3\} \in \mathcal{T}$ and there is no subset $\mathcal{A}$ in $\mathfrak{I}$ such that $\mathcal{F}(\mathcal{A}) = \{3\}$.

Definition 3.5: A mapping $\mathcal{F}: (\mathbb{X}, \mathfrak{I}_{\mathbb{X}}) \to (\mathbb{Y}, \mathcal{T}_{\mathbb{Y}})$ is named an *ideal correspondence* if $\mathcal{F}$ is ideal contrast and ideal exhaustive.

Theorem 3.6: If $\mathcal{F}: (\mathbb{X}, \mathfrak{I}_{\mathbb{X}}) \to (\mathbb{Y}, \mathcal{T}_{\mathbb{Y}})$ is ideal correspondence, implies $\mathcal{F}(\mathfrak{I}_{\mathbb{X}}) = \mathcal{T}_{\mathbb{Y}}$.

Proof: Let $\mathcal{F}(\mathcal{A}) \in \mathcal{F}(\mathfrak{I}_{\mathbb{X}})$ then $\in \mathfrak{I}_{\mathbb{X}}$. but $\mathcal{F}$ is an ideal correspondence then $\mathcal{F}$ is ideal contrast. So, there exist $\mathcal{B} \in \mathcal{T}_{\mathbb{Y}}$,where $\mathcal{F}(\mathcal{A}) = \mathcal{B}$ which implies that, $\mathcal{F}(\mathcal{A}) \in \mathcal{T}_{\mathbb{Y}}$. That is $\mathcal{F}(\mathfrak{I}_{\mathbb{X}}) \subset \mathcal{T}_{\mathbb{Y}}$.

Conversely, let $\mathcal{B} \in \mathcal{T}$ since $\mathcal{F}$ is ideal totality, then there exist $\mathcal{A} \in \mathfrak{I}$ for which, $\mathcal{F}(\mathcal{A}) = \mathcal{B}$ but $\mathcal{F}(\mathcal{A}) \in \mathcal{F}(\mathfrak{I})$ then $\mathcal{B}$ in $\mathcal{F}(\mathfrak{I})$. Hence, $\mathcal{T} \subset \mathcal{F}(\mathfrak{I})$.

Thus, $\mathcal{F}(\mathfrak{I}) = \mathcal{T}$.

Definition 3.7: A function $\oint\!\!\!\!\oint : (\mathbb{X}, \mathfrak{I}_{\mathbb{X}}) \to (\mathbb{Y}, \mathcal{T}_{\mathbb{Y}})$ is named *ideal continuous* if for any $x \in \mathcal{X}$ and for any $\mathcal{T}_{\mathbb{Y}}$-set $\mathcal{B}$ such that, $\oint\!\!\!\!\oint(x) \in \mathcal{B}$, there exist $\mathfrak{I}_{\mathbb{X}}$-set $\mathcal{A}$ such that $x \in \mathcal{A}$ and $\mathcal{B} \subset \oint\!\!\!\!\oint(\mathcal{A})$.

Theorem 3.8: In the bijective ideal continuous mapping $\mathcal{F}: (\mathbb{X}, \mathfrak{I}_{\mathbb{X}}) \to (\mathbb{Y}, \mathcal{T}_{\mathbb{Y}})$, the inverse image of any $\mathcal{T}_{\mathbb{Y}}$-set is $\mathfrak{I}_{\mathbb{X}}$ -set.

Proof: Let $\mathcal{F}$ be onto ideal continuous mapping and$\mathcal{B} \in \mathcal{T}_{\mathbb{Y}}$.

If $\mathcal{B} = \emptyset$, then $\mathcal{F}^{-1}(\mathcal{B}) = \emptyset$ which is an $\mathfrak{I}_{\mathbb{X}}$-set. If $\mathcal{B} \neq \emptyset$ implies for any $y \in \mathcal{B}$, but $\mathcal{F}$ is surjective , we can find $x\ in\ \mathcal{X}$ where $y = \mathcal{F}(x)$, but $\mathcal{F}^{-1}(y) \in \mathcal{F}^{-1}(\mathcal{B})$ so, $\mathcal{F}(x) \in \mathcal{B}$. Now, but $\mathcal{F}$ is ideal continuous, $x \in \mathcal{X}$ and $\mathcal{F}(x) \in \mathcal{B}$, hence there exist $H \in \mathfrak{I}_{\mathbb{X}}$ such that $x \in H$ and $\mathcal{B} \subset \mathcal{F}(H)$ implies $\mathcal{F}^{-1}(\mathcal{B}) \subset H$ and by assume $\mathfrak{I}_{\mathbb{X}}$ is an ideal, we get that, $\mathcal{F}^{-1}(\mathcal{B})$ is $\mathfrak{I}_{\mathbb{X}}$ -set.

Definition 3.9: Let $\oint\!\!\!\!\oint : (\mathbb{X}, \mathfrak{I}_{\mathbb{X}}) \to (\mathbb{Y}, \mathcal{T}_{\mathbb{Y}})$ be a function, we say that $\oint\!\!\!\!\oint$ is *ideal homeomorphism* if $\mathcal{F}$ is bijective and each of $\oint\!\!\!\!\oint$, $\oint\!\!\!\!\oint\, \mathcal{F}^{-1}$ is an ideal continuous.

Theorem 3.10: A one to one and onto mapping $\mathcal{F}: (\mathbb{X}, \mathfrak{I}_{\mathbb{X}}) \to (\mathbb{Y}, \mathcal{T}_{\mathbb{Y}})$ is ideal homeomorphism if $\mathcal{F}$ is ideal correspondence.

Proof: Let $\mathcal{F}$ is ideal homeomorphism, to prove that $\mathcal{F}$ is ideal correspondence, let $\mathcal{A}$ be any $\mathfrak{I}_{\mathbb{X}}$ -set, so, because $\mathcal{F}^{-1} : (\mathbb{Y}, \mathcal{T}_{\mathbb{Y}}) \to (\mathbb{X}, \mathfrak{I}_{\mathbb{X}})$ is ideal continuous, then $(\mathcal{F}^{-1})^{-1}(\mathcal{A}) \in \mathcal{T}_{\mathbb{Y}}$. But, $\mathcal{F}$ is injective and onto then $\mathcal{F}(\mathcal{A}) = (\mathcal{F}^{-1})^{-1}(\mathcal{A})$ and hence, there exist $\mathcal{B} \in \mathcal{T}_{\mathbb{Y}}$ such that $\mathcal{F}(\mathcal{A}) = \mathcal{B}$. Hence, we obtain that $\mathcal{F}$ is ideal contrast.

Now, let $\mathcal{B}$ is $\mathcal{T}_{\mathbb{Y}}$-set, but $\mathcal{F}$ is ideal continuous, then by theorem 3.8, $\mathcal{F}^{-1}(\mathcal{B})$ is $\mathfrak{I}_{\mathbb{X}}$ -set. Let $\mathcal{F}^{-1}(\mathcal{B}) = \mathcal{A}$ for some $\mathfrak{I}_{\mathbb{X}}$ -set $\mathcal{A}$, by assume $\mathcal{F}$ is injective and onto then $\mathcal{F}(\mathcal{A}) = \mathcal{F}\big(\mathcal{F}^{-1}(\mathcal{B})\big) = \mathcal{B}$, so $\mathcal{F}$ is ideal exhaustive.

Conversely: Let $\mathcal{F}$ is ideal correspondence to prove $\mathcal{F}$ is ideal homeomorphism, we need to prove $\mathcal{F}$ and $\mathcal{F}^{-1}$ are ideal continuous.

Let $x \in \mathcal{X}$ and $\mathcal{B} \in \mathcal{T}_{\mathbb{Y}}$ such that $\mathcal{F}(x) \in \mathcal{B}$. To prove that there exist $\mathcal{A} \in \mathfrak{I}_{\mathbb{X}}$, $x \in \mathcal{A}$ and $\mathcal{B} \subset \mathcal{F}(\mathcal{A})$.

By assume $\mathcal{F}$ is ideal exhaustive, we have $\mathfrak{I}_{\mathbb{X}}$ -set U in $\mathcal{X}$ satisfy $\mathcal{F}(\mathcal{A}) = \mathcal{B}$, if $x \notin \mathcal{A}$, then $\mathcal{F}(x) \notin \mathcal{F}(\mathcal{A})$ that is, $\mathcal{F}(x) \notin \mathcal{B}$ which is a contradiction. Hence, $x \in \mathcal{A}$.

Thus, $\mathcal{F}$ is ideal continuous.

To prove $\mathcal{F}^{-1} : (\mathbb{Y}, \mathcal{T}_{\mathbb{Y}}) \to (\mathbb{X}, \mathfrak{I}_{\mathbb{X}})$is ideal continuous. Let $y \in \Upsilon$ and $\mathcal{A} \in \mathfrak{I}_{\mathbb{X}}$ such that, $\mathcal{F}^{-1}(y) \in \mathcal{A}$. Since $\mathcal{F}$ is ideal contrast then there is a $\mathcal{T}_{\mathbb{Y}}$-set $\mathcal{B}$ in Υ such that $\mathcal{F}(\mathcal{A}) = \mathcal{B}$ and hence, $\mathcal{F}^{-1}(\mathcal{F}(\mathcal{A})) = \mathcal{F}^{-1}(\mathcal{B})$. but $\mathcal{F}$ is one to one and onto, then $\mathcal{A} = \mathcal{F}^{-1}(\mathcal{B})$.

If $\mathrm{y} \notin \mathcal{B}$, hence $\mathcal{F}^{-1}(\mathrm{y}) \notin \mathcal{F}^{-1}(\mathcal{B})$and then $\mathcal{F}^{-1}(\mathrm{y}) \notin \mathcal{A}$ which is contradiction.

Then $\mathcal{F}^{-1}(y) \in \mathcal{F}^{-1}(\mathcal{B})$ that is, $y \in \mathcal{B}$

So there is a $\mathcal{J}_{\mathbb{Y}}$-set $\mathcal{B}$ in Υ, $y \in \mathcal{B}$ and $\mathcal{F}^{-1}(\mathcal{B}) = \mathcal{A}$. Hence, $\mathcal{F}^{-1}$ is ideal continuous.

Thus, $\mathcal{F}$ is ideal homeomorphism.

Proposition 3.11: Let $\mathcal{F}: (\mathbb{X}, \mathfrak{T}_{\mathbb{X}}) \to (\mathbb{Y}, \mathcal{J}_{\mathbb{Y}})$ is injective correspondence, implies $\mathcal{F}^{-1}(\mathcal{J}_{\mathbb{Y}}) = \mathfrak{T}_{\mathbb{X}}$

Proof: - Let $\mathcal{F}^{-1}(\mathcal{B}) \in \mathcal{F}^{-1}(\mathcal{J}_{\mathbb{Y}})$ then $\mathcal{B} \in \mathcal{J}_{\mathbb{Y}}$, but $\mathcal{F}$ is ideal totalitarianism, then we can find $\mathcal{A} \in \mathfrak{T}_{\mathbb{X}}$,where $\mathcal{B} = \mathcal{F}(\mathcal{A})$. By assume $\mathcal{F}$ is injective then $\mathcal{F}^{-1}(\mathcal{B}) = \mathcal{F}^{-1}\big(\mathcal{F}(\mathcal{A})\big) = \mathcal{A}$,hence, $\mathcal{F}^{-1}(\mathcal{B}) \in \mathfrak{T}_{\mathbb{X}}$. That is, $\mathcal{F}^{-1}(\mathcal{J}_{\mathbb{Y}}) \subset \mathfrak{T}_{\mathbb{X}}$.

Now, let $\mathcal{A} \in \mathfrak{T}_{\mathbb{X}}$ and since $\mathcal{F}$ is ideal contrast then there exists $\mathcal{B} \in \mathcal{J}_{\mathbb{Y}}$, such that $\mathcal{B} = \mathcal{F}(\mathcal{A})$. By assume $\mathcal{F}$ is injective then $\mathcal{F}^{-1}(\mathcal{B}) = \mathcal{F}^{-1}(\mathcal{F}(\mathcal{A}) = \mathcal{A}$.

Since $\mathcal{B} \in \mathcal{J}_{\mathbb{Y}}$ then $\mathcal{F}^{-1}(\mathcal{B}) \in \mathcal{F}^{-1}(\mathcal{J}_{\mathbb{Y}})$ and hence, $\mathcal{A} \in \mathcal{F}^{-1}(\mathcal{J}_{\mathbb{Y}})$. That is, $\mathfrak{T}_{\mathbb{X}} \subset \mathcal{F}^{-1}(\mathcal{J}_{\mathbb{Y}})$. Thus, $\mathcal{F}^{-1}(\mathcal{J}_{\mathbb{Y}}) = \mathfrak{T}_{\mathbb{X}}$.

4. Discussion

The outcome of this research will give a common language for describing the connections between ideal spaces via the connection between their corresponding ideal subsets. The ideal mapping that occurs in the direction of ideal contrast and ideal exhaustive mapping are complementary directions of interaction between the ideal structure of the domain and the ideal structure of the codomain, and a combination of these two directions of interaction defines the ideal correspondence mapping. This method gives a more comprehensive structural view of the mappings between ideal spaces other than merely continuity and complements the earlier studies on the generalized continuity and mapping properties in ideal topological spaces [15,17, 20].

The findings related to the ideal continuity provide an important link between the concepts of mapping introduced and the current continuity structures. In the ideal topological spaces, generalized continuity, fuzzy ideal continuity and related continuity classes are very important concepts, which have been highlighted recently [15,17,20]. In this paper, Theorem 3.8 proves, under the above bijective conditions, that the inverse image of an ideal subset is also an ideal subset. This result is one of the first that establishes a direct link between the proposed mapping framework and the behavior of ideal subsets under continuous mappings.

One of the main findings of this study is that of Theorem 3.10, which relates ideal correspondence and ideal homeomorphism in one-to-one and onto mappings. The result shows that the two direction properties which make an ideal correspondence do is necessary condition to give the structural conditions needed for an ideal homeomorphism. This is a continuation of the general theme of Continuity and Homeomorphism-type in ideal topological spaces [15,17,20]. Proposition 3.11 also gives more conditions on the relationships between the corresponding ideal subsets for being injective.

The proposed framework is also in line with recent developments on generalized open sets [8, 10, 12, 14] and set-operators [9, 11, 13] and continuity structures in ideal topological spaces [10, 12, 13, 14]. These studies illustrate the possibility of building and describing generalized topological properties by means of relations between sets of ideals. The theme of the present work is instead the one of the directions between ideal subsets, based on the new concepts of ideal contrast, ideal exhaustive and ideal correspondence mappings.

The results indicate, overall, that ideal contrast, ideal exhaustive and ideal correspondence mappings are complementary tools to study the relationship between ideal spaces. The proposed framework can provide a starting point for the study of other homeomorphism-type properties of compositions, generalized continuity and others for ideal topological spaces.

5. Conclusion

Three types of mappings were introduced and studied in this study; ideal contrast, ideal exhaustive and ideal correspondence mappings. These results show that these ideas are complementary in describing the relationships between the ideal subsets of the domain and codomain. Specifically, the notion of ideal exhaustive is combined with the notion of ideal contrast in the notion of ideal correspondence.

The study also introduced relationships between the concepts of ideal correspondence, ideal continuity and ideal homeomorphism. It is observed that the new mapping properties could be used to describe some important structural relationships between ideal spaces, in suitable injective, surjective and bijective cases. In particular, it is proved in Theorem 3.10 that a bijective mapping is an ideal homeomorphism which satisfies ideal correspondence property, thus a direct relation between the newly introduced mapping concept and ideal homeomorphism is provided.

The proposed structure is an extension of the investigation of mappings in ideal topological spaces, but has a more direct look at the relationship between the ideal structures associated with the mapping. The results are related to the ones obtained in previous research performed in the area of generalized continuity, ideal open sets and related operators [8–20]. The results obtained will serve as a foundation for further research in the field of generalized mappings, compositions, continuity properties and homeomorphism-type structures in ideal spaces.

References

1. Casimir Kuratowski. Topologie I. Instytut Matematyczny Polskiej Akademi Nauk, (1933).
2. R. Vaidyanathaswamy. The localization theory in set-topology. In Proceedings of the Indian Academy of Sciences-Section A, volume 20, pages 51–61. Springer India New Delhi, (1944).
3. Dragan Jankovic and TR Hamlett. New topologies from old via ideals. The American Mathematical Monthly, 97(4):295–310, (1990).
4. DS Jankovic. On semi separation properties. Indian J. Pure Appl. Math., 16(9):957–964, (1985).
5. Eiichi Hayashi. Topologies defined by local properties. Mathematische Annalen, 156(3):205–215, (1964).
6. T.R Hamlett. Compactness with respect to an ideal. Boll. Un. Mat. Ital. (7), 4:849–861, (1990).
7. Boonpok, C., & Sama-Ae, A. (2026). Submaximality, connectedness, and δβI-continuous mappings in ideal spaces. *Mathematics, 14*(14), 2656.DOI: 10.3390/math14142656.

8. Almuhur, E., Al-Labadi, M., & Bashir, R. (2026). hα-Almost weakly continuous functions in ideal topological spaces. *International Journal of Analysis and Applications, 24*, 230.DOI: 10.28924/2291-8639-24-2026-230.
9. Almuhur, E., Al-Labadi, M., Bashir, R., Qoqazeh, H., & Audeh, W. (2026). Further properties of hα-open sets in ideal topological spaces. *International Journal of Analysis and Applications, 24*, 181.DOI: 10.28924/2291-8639-24-2026-181.
10. Kong-ied, B., Sama-Ae, A., & Boonpok, C. (2026). A unified theory of near continuity for functions defined between an ideal topological space and a bitopological space. *International Journal of Analysis and Applications, 24*, 82. DOI: 10.28924/2291-8639-24-2026-82.
11. Güldürdek, A., & Açıkgöz, A. (2026). Two idealizations of h-open sets: Interior operator and continuity. *Mathematics, 14*(16), 2861.DOI: 10.3390/math14162861.
12. Abbas, S. E., El-Sanowsy, E. S., Khiamy, H., & Ibedou, I. (2025). A new set-operator on ideal topological spaces. *Bulletin of the International Mathematical Virtual Institute, 15*(1), 85–94.DOI: 10.7251/BIMVI2501085A.
13. Özen Yıldırım, S., & Tunç, A. N. (2025). hΓ-open sets in ideal topological spaces. *Journal of New Theory*, 50, 14–23.DOI: 10.53570/jnt.1747410.
14. Sereti, F. (2024). A study of new dimensions for ideal topological spaces. *Applied General Topology*.DOI: 10.4995/agt.2024.19760.
15. Alsharari, F. (2024). Fuzzy (Almost, δ) ideal continuous mappings. *International Journal of Analysis and Applications, 22*, 59.DOI: 10.28924/2291-8639-22-2024-59.
16. AL-Jaheishi, R. I., & Al-Siraj, A. (2024). Application of ic-open sets in topologies via ideals. *AL-Rafidain Journal of Computer Sciences and Mathematics, 18*(1), 92–95.DOI: 10.33899/csmj.2024.146288.1104.
17. Njamcul, A., & Pavlović, A. (2023). Various types of continuity and their interpretations in ideal topological spaces. *Turkish Journal of Mathematics, 47*(7), 2086–2097.DOI: 10.55730/1300-0098.3482.
18. Jadoo, M. W., & Alsaraj, A. A. M. (2023). Some classes in ideal topological spaces. *Journal of Education and Science, 32*(2), 128–133.DOI: 10.33899/edusj.2023.137766.1318.
19. Mahmood, P. M. (2023). On contra b-I-continuous and contra b-I-open functions. *Tikrit Journal of Pure Science, 20*(2), 177–186.DOI: 10.25130/tjps.v20i2.1178.
20. Abdulraouf, A. E., & Abd Al-Zhara, S. A. (2022). On some kinds of continuous functions in ideal topological space. *Journal of Interdisciplinary Mathematics, 25*(6), 1885–1892.DOI: 10.1080/09720502.2022.2046342