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### Yang Transformation for Solving Transport and Poisson Equations

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#### Abstract

Integral Transforms are analytical tools used to solve differential and partial equation. In this paper, the yang transform is applied to solve the Poisson and transport equations. And the general solution formula of the transport equation was deduced in both homogenous and non- homogenous cases under initial and boundary conditions. And also finding the general solution formula for the Poisson equation. This method has proven that yang transform is an effective tool with high efficiency in solving complex equations and transforming in to simpler algebraic equations, it can be extended to handle other more complex mathematical problems indifferent mathematical fields in engineering and applied sciences. And a set of multiple illustrative examples was presented to prove the efficiency and accuracy of the deduced formulas.

#### Keywords

Transformations,  
New Integral Transform  
Transport,  
Poisson Equations



## 1-Introduction

Integral transforms are an effective tool for solving differential equations (7).

They are commonly used in mechanics, astronomy, physics, and many other fields [1,5]. The origin of integral transforms—particularly the Laplace and Fourier transforms—traces back to the scholar Laplace (1749-1827) in the 1780s; these methods, grounded in rigorous mathematics, are on par with modern operational calculus [6].

There are different types of integral transformations, including the Laplace, Temimi, Elzaki, Sumudu and Shehu, [3,4,10,9,8]

The Yang transform is anew transformation emerged used to solve some type of

In this of the modern transforms is the Yang transform, used to solve certain types of differential equations and applied in physics and engineering, the Yang transform is applied to Transport and Poisson equations.

This work, we address the solution to the Poisson and transport equations. The second section covers certain definitions and properties; section3 derives solution formulas and the final section includes the solutions to specific examples.

## 2-Definition [13]

The Yang transform of  $q(\tau)$ , is defined for an is integrable function  $q(\tau)$ ,  $\tau > 0$  as:

$$Y_1(q(\tau)) = \tau(v) = \int_0^\infty e^{-\frac{\tau}{v}} q(\tau) dt, \quad t > 0 \quad (2.1)$$

Where the integral exists for some  $v$ , where  $v \in (-\tau_1, \tau_2)$ .

If we substitute  $\frac{-\tau}{v} = \mu$  then equation (2.1) becomes,

$$Y_1(q(\tau)) = \tau(v) = v \int_0^\infty e^{-\mu} q(v\mu) d\mu, \quad \mu > 0 \quad (2.2)$$

## 3 - Preliminaries Yang transform.

**Property (3.1):** if  $\omega_1(\delta), \omega_2(\delta), \dots, \omega_n(\delta)$  have Yang transform, then

$$Y_1(\beta_1 \omega_1(\delta) + \beta_2 \omega_2(\delta) + \dots + \beta_n \omega_n(\delta)) = \beta_1 Y_1(\omega_1(\delta)) + \beta_2 Y_1(\omega_2(\delta)) + \dots + \beta_n Y_1(\omega_n(\delta)) \quad (3.1)$$

where  $\beta_1, \beta_2, \dots, \beta_n$  are constants  $\omega_1(\delta), \omega_2(\delta), \dots, \omega_n(\delta)$  are defined function.

**Theorem (3.2):** Let  $J(Y, \alpha)$  be a function defined for  $\alpha > 0$  and  $Y \in [\sigma, \rho]$ . Then

$$(i) Y_1\left(\frac{\partial J}{\partial \alpha}\right) = \frac{1}{\alpha} \vartheta - J(Y, 0), \quad \text{where } \vartheta = \vartheta(Y, \epsilon) = Y_1\{J(Y, \alpha)\}$$

$$(ii) Y_1\left(\frac{\partial J}{\partial \alpha}\right) = \frac{\partial \vartheta}{\partial \alpha}$$

$$(iii) \frac{T(Y, \alpha)}{\alpha^2} - \frac{1}{\alpha} J(\alpha, 0) - J_r(\alpha, 0) + \frac{d^2}{dY^2} T(Y, \alpha) = Y_1(G(Y, r)),$$

#### 4-Table of Selected Yang transformation [13]:

The Yang Transformation was shown in the table below for the following functions:

| ID | function $\beta(\tau)$ | $Y(\beta(\tau))$                              |
|----|------------------------|-----------------------------------------------|
| 1  | 1                      | $\partial$                                    |
| 2  | $\mu$                  | $\partial^2$                                  |
| 3  | $\mu^\eta$             | $\eta! \cdot \partial^{\eta+1}$               |
| 4  | $e^{\rho\mu}$          | $\frac{\partial}{1 - \partial\rho}$           |
| 5  | $\sin \rho\mu$         | $\frac{\rho\partial^2}{1 + \rho^2\partial^2}$ |
| 6  | $\cos \rho\mu$         | $\frac{\partial}{1 + \rho^2\partial^2}$       |
| 7  | $\sinh \rho\mu$        | $\frac{\rho\partial^2}{1 - \rho^2\partial^2}$ |
| 8  | $\cosh \rho\mu$        | $\frac{\partial}{1 + \rho^2\partial^2}$       |

#### 5 -Yang Transformation

In this section, Yang transformation is used to derive solution formulas:

##### Formula (4.1):

Consider the following equation for homogeneous Transport equation: -

$$I_r + n I_Y = 0, \quad (4.1) \text{ under the conditions}$$

$$I(Y, 0) = q(Y),$$

$$I(0, r) = I(1, r) = 0.$$

and  $n$  is constant.

By taking Yang transformation to both sides:

$$\frac{T(Y, \alpha)}{\alpha} - I(Y, 0) + n \frac{d}{dY} T(Y, \alpha) = 0,$$

After applying the initial conditions:

$$\frac{d}{dY} T(Y, \alpha) + \frac{1}{\alpha n} T(Y, \alpha) = \frac{1}{n} q(Y), \quad (4.2)$$

The above equation (4.2) is an ordinary linear differential equation and has a solution.

$$T(Y, \alpha) e^{\frac{1}{n\alpha} Y} = \frac{1}{n} \int q(Y) \cdot e^{\frac{1}{n\alpha} r} dr$$

$$T(Y, \alpha) = \frac{1}{n} e^{-\frac{1}{n\alpha} Y} \int q(Y) \cdot e^{\frac{1}{n\alpha} r} dr,$$

Thus, the solution for (4. 1), by taking the inverse of Yang transformation.

$$L(Y, r) = Y_1^{-1} \left[ \frac{1}{n} e^{-\frac{1}{n\alpha} Y} \int q(Y) e^{\frac{1}{n\alpha} r} dr \right]. \quad (4.3)$$

### Formula (4.2):

Consider the following equations for non-homogeneous Transport equation:-

$$l_r + n l_Y = G(Y, r), \quad (4.4)$$

under the conditions  $l(Y, 0) = q(Y)$ ,

$$l(0, r) = l(1, r) = 0.$$

and  $n$  is constant.

By taking Yang transformation to both sides:

$$\frac{T(Y, \alpha)}{\alpha} - l(Y, 0) + n \frac{d}{dY} T(Y, \alpha) = Y_l(G(Y, r)),$$

After substituting the initial conditions: -

$$\frac{d}{dY} T(Y, \alpha) + \frac{1}{\alpha n} T(Y, \alpha) = \frac{1}{n} Y_l(G(Y, r)) + \frac{1}{n} q(Y). \quad (4.5) \text{ The above equation (4. 5) is}$$

an ordinary linear differential equation and has a solution.

$$T(Y, \alpha) e^{\frac{-1}{n\alpha} Y} = \frac{1}{n} \int [Y_l(G(Y, r)) + q(Y)] e^{\frac{1}{n\alpha} r} dr,$$

$$T(Y, \alpha) = \frac{1}{n} e^{\frac{-1}{n\alpha} Y} \int \{Y_l(G(Y, r)) + q(Y)\} \cdot e^{\frac{1}{n\alpha} r} dr,$$

we get the general solution of equation (4.5), by taking the inverse of Yang transformation.

$$T(Y, \alpha) = Y_l^{-1} \left[ \frac{1}{n} e^{\frac{-1}{n\alpha} Y} \int Y_l(G(Y, r)) \cdot e^{\frac{1}{n\alpha} r} dr + \frac{1}{n} e^{\frac{1}{n\alpha} r} \int q(Y) e^{\frac{1}{n\alpha} r} dr \right]$$

### Formula (4.3):

Consider Poisson equation is: -

$$l_{rr}(Y, r) + l_{YY}(Y, r) = G(Y, r), \quad (4.6)$$

sol  $l(Y, 0) = q(Y)$ ,  $l_r(Y, 0) = h(Y)$ ,

and boundary conditions  $l(0, r) = l(1, r) = 0$ .

By taking Yang transformation to both sides:

$$\frac{T(Y, \alpha)}{\alpha^2} - \frac{1}{\alpha} l(\alpha, 0) - l_r(\alpha, 0) + \frac{d^2}{dY^2} T(Y, \alpha) = Y_l(G(Y, r)),$$

After substituting the initial conditions:

$$\frac{d^2}{dY^2} T(Y, \alpha) + \frac{T(Y, \alpha)}{\alpha^2} = Y_l(G(Y, r)) + \frac{1}{\alpha} q(Y) + h(Y). \quad (4.7)$$

The above equation (4.7) is an ordinary linear differential equation which has a complementary solution.

$$T_c(Y, \alpha) = p_1 \cos\left(\frac{1}{\alpha}\right) Y + p_2 \sin\left(\frac{1}{\alpha}\right) Y.$$

where  $p_1$  and  $p_2$  are an arbitrary constant.

Using variation of parameters, then particular solution is:

$$T_p(Y, \alpha) = p_1(Y) \cos\left(\frac{1}{\alpha}\right) Y + p_2(Y) \sin\left(\frac{1}{\alpha}\right) Y.$$

Since  $Y_l(G(Y, r)) + \frac{1}{\alpha} q(Y) + h(Y)$

$$p_1'(Y) \cos\left(\frac{1}{\alpha}\right) Y + p_2'(Y) \sin\left(\frac{1}{\alpha}\right) Y = 0,$$

$$- \frac{1}{\alpha} p_1(Y) \sin\left(\frac{1}{\alpha}\right) Y + \frac{1}{\alpha} p_2(Y) \cos\left(\frac{1}{\alpha}\right) Y = Y_l(G(Y, r)) + \frac{1}{\alpha} q(Y) + h(Y),$$

Where:

$$\nabla = \begin{vmatrix} \cos\left(\frac{1}{\alpha}\right)Y & \sin\left(\frac{1}{\alpha}\right)Y \\ -\frac{1}{\alpha}\sin\left(\frac{1}{\alpha}\right)Y & \frac{1}{\alpha}\cos\left(\frac{1}{\alpha}\right)Y \end{vmatrix} = \frac{1}{\alpha}.$$

$$\begin{aligned} p_1(Y) &= \frac{1}{\nabla} \begin{vmatrix} 0 & \sin\left(\frac{1}{\alpha}\right)Y \\ \gamma_1(G(Y, r)) + \frac{1}{\alpha}q(Y) + h(Y) & \frac{1}{\alpha}\cos\left(\frac{1}{\alpha}\right)Y \end{vmatrix} \\ &= \frac{-[\gamma_1(G(Y, r)) + \frac{1}{\alpha}q(Y) + h(Y)](\sin\left(\frac{1}{\alpha}\right)Y)}{\frac{1}{\alpha}}, \end{aligned}$$

then

$$p_1(Y) = -\alpha \int [\gamma_1(G(Y, r)) + \frac{1}{\alpha}q(Y) + h(Y)](\sin\left(\frac{1}{\alpha}\right)Y) dY.$$

In the same way:

$$p_2(Y) = \frac{(\gamma_1(G(Y, r)) + \frac{1}{\alpha}q(Y) + h(Y))\cos\left(\frac{1}{\alpha}\right)Y}{\frac{1}{\alpha}},$$

$$p_2(Y) = \alpha \int \left( (\gamma_1(G(Y, r)) + \frac{1}{\alpha}q(Y) + h(Y)) \cos\left(\frac{1}{\alpha}\right)Y \right) dY.$$

Equation (4.7) yields the general solution:

$$\begin{aligned} T(Y, \alpha) &= [p_1 \cos\left(\frac{1}{\alpha}\right)Y + p_2 \sin\left(\frac{1}{\alpha}\right)Y] + \\ &\quad \left[ \left( -\alpha \int (\gamma_1(G(Y, r)) + \frac{1}{\alpha}q(Y) + h(Y)) \sin\left(\frac{1}{\alpha}\right)Y dY \right) \cos\left(\frac{1}{\alpha}\right)Y + \right. \\ &\quad \left. \left( \alpha \int ((\gamma_1(G(Y, r)) + q(Y) + h(Y))) \cos\left(\frac{1}{\alpha}\right)Y dY \right) \sin\left(\frac{1}{\alpha}\right)Y \right], \end{aligned}$$

utilizing the boundary conditions, yields:

$$p_1 = p_2 = 0$$

$$T(Y, \alpha) = \left[ \begin{aligned} & \left( -\alpha \int (\gamma_1(G(Y, r)) + q(Y) + h(Y)) \sin\left(\frac{1}{\alpha}\right)Y dY \right) \cos\left(\frac{1}{\alpha}\right)Y + \\ & \left( \alpha \int ((\gamma_1(G(Y, r)) + \frac{1}{\alpha}q(Y) + h(Y))) \cos\left(\frac{1}{\alpha}\right)Y dY \right) \sin\left(\frac{1}{\alpha}\right)Y \end{aligned} \right]$$

from (4.6), by taking the inverse of Yang transformation.

$$\begin{aligned} L(x, \mu) &= \gamma_1^{-1} \left[ \left( -\alpha \int \left( (G(Y, r)) + \frac{1}{\alpha}q(Y) + h(Y) \right) \sin\left(\frac{1}{\alpha}\right)Y dY \right) \right. \\ &\quad + \left( \alpha \int \left( (\gamma_1(G(Y, r)) + \frac{1}{\alpha}q(Y) \right. \right. \\ &\quad \left. \left. + h(Y)) \cos\left(\frac{1}{\alpha}\right)Y dY \right) \sin\left(\frac{1}{\alpha}\right)Y \right]. \end{aligned} \quad (4.8)$$

**Example (5.1):**

To solve the homogeneous transport equation:-

$$\mu_Y + 3 \mu_r = 0 \quad (5.1)$$

under the conditions  $l(Y, 0) = e^Y$  and  $l(0, r) = l(1, r) = 0$ .

sol:

Using the equation (4.3) with the for formula (4.1)

$$I(Y, r) = Y_1^{-1} \left[ \frac{1}{n} e^{\frac{-1}{n\alpha} Y} \int q(Y) e^{\frac{1}{n\alpha} r} dr \right]$$

substitute the give function:-

$$= Y_1^{-1} \left[ 3e^{\frac{-3}{\alpha} r} \int e^Y \cdot e^{\frac{3}{\alpha} r} dr \right]$$

$$= Y_1^{-1} \left[ 3e^{\frac{-3}{\alpha} r} \int e^{\left(1+\frac{3}{\alpha}\right)r} dr \right]$$

After simplified and integration:

$$= Y_1^{-1} \left[ \frac{3\alpha}{3+\alpha} e^r \right]$$

$$= e^r Y_1^{-1} \left[ \frac{\alpha}{1+\frac{\alpha}{3}} \right]$$

$$I(Y, r) = T(Y, r) = e^r e^{\frac{-1}{3}r}.$$

### Example (5.2):

To solve the non- homogeneous transport equation: -

$$2 \mu_Y + \mu_r = 6 \quad (5.2)$$

under the conditions  $I(Y, 0) = e^{2Y}$  and  $I(0, r) = I(1, r) = 0$

sol:

Using the equation (4.6) with the for formula (4.2)

$$I(Y, r) = Y_1^{-1} \left[ \frac{1}{n} e^{\frac{-1}{n\alpha} Y} \int Y_1 G(Y, r) \cdot e^{\frac{1}{n\alpha} r} dr + \frac{1}{n} e^{\frac{-1}{n\alpha} Y} \int q(Y) \cdot e^{\frac{1}{n\alpha} r} dr \right]$$

substitute the give function: -

$$= Y_1^{-1} \left[ \frac{1}{2} e^{\frac{-1}{2\alpha} r} \int 6\alpha^2 \cdot e^{\frac{1}{2\alpha} r} dr + \frac{1}{2} e^{\frac{-1}{2\alpha} r} \int e^{2Y} \cdot e^{\frac{1}{2\alpha} r} dr \right]$$

After, simplified and integration:

$$= Y_1^{-1} \left[ 6\alpha^2 + \frac{1}{2} e^{\frac{-1}{2\alpha} r} \cdot \frac{1}{\left(2+\frac{1}{2\alpha}\right)} e^{\left(2+\frac{1}{2\alpha}\right)r} \right]$$

$$= Y_1^{-1} \left[ 6\alpha^2 + \frac{\alpha}{1+4\alpha} e^{2r} \right]$$

$$= 6Y_1^{-1}[\alpha^2] + e^{2r} Y_1^{-1} \left[ \frac{\alpha}{1+4\alpha} \right]$$

$$I(Y, r) = 6t + e^{2r} e^{-4t}.$$

Which represent the particular solution of equation (5.2).

**Example (5.3):**

To solve the Poisson equation

$$\mathcal{I}_{rr}(Y, r) + \mathcal{I}_{YY}(Y, r) = e^r \quad (5.3)$$

under the conditions  $\mathcal{I}(Y, 0) = 0$ ,  $\mathcal{I}_r(Y, 0) = 1$ ,

$$\text{and } \mathcal{I}(0, r) = \mathcal{I}(1, r) = 0,$$

sol

$$\mathcal{I}(Y, r) = Y_1^{-1} \left[ \frac{1}{n} e^{\frac{-1}{n\alpha} Y} \int Y_1 G(Y, r) \cdot e^{\frac{1}{n\alpha} r} dr + \frac{1}{n} e^{\frac{-1}{n\alpha} Y} \int q(Y) \cdot e^{\frac{1}{n\alpha} r} dr \right]$$

Using the equation (4.8) with the formula (4.3)

$$\begin{aligned} T(Y, r) = Y_1^{-1} & \left( -\alpha \int \left( Y_1(G(Y, r)) + \frac{1}{\alpha} q(Y) + f_1(Y) \right) \sin\left(\frac{1}{\alpha} Y\right) dY \right) \cos\left(\frac{1}{\alpha} Y\right) \\ & + \left( \alpha \int \left( Y_1(G(Y, r)) + q(Y) + f_1(Y) \right) \cos\left(\frac{1}{\alpha} Y\right) dY \right) \sin\left(\frac{1}{\alpha} Y\right) \end{aligned}$$

Substituting the given function, yield:

$$\begin{aligned} &= Y_1^{-1} \left[ \left( -\alpha \int \left( \frac{\alpha}{1-\alpha} + 1 \right) \sin\left(\frac{1}{\alpha} Y\right) dY \right) \cos\left(\frac{1}{\alpha} Y\right) \right. \\ &\quad \left. + \left( \alpha \int \left( \frac{\alpha}{1-\alpha} + 1 \right) \cos\left(\frac{1}{\alpha} Y\right) dY \right) \sin\left(\frac{1}{\alpha} Y\right) \right] \\ &= Y_1^{-1} \left[ \left( \left( \frac{-\alpha}{1-\alpha} \right) \int \sin\left(\frac{1}{\alpha} Y\right) dY \right) \cos\left(\frac{1}{\alpha} Y\right) + \left( \left( \frac{\alpha}{1-\alpha} \right) \int \cos\left(\frac{1}{\alpha} Y\right) dY \right) \sin\left(\frac{1}{\alpha} Y\right) \right] \end{aligned}$$

After, simplified and integration:

$$\begin{aligned} &= Y_1^{-1} \left[ \frac{\alpha}{1-\alpha} \cos^2\left(\frac{1}{\alpha} Y\right) + \frac{\alpha}{1-\alpha} \sin^2\left(\frac{1}{\alpha} Y\right) \right] \\ &= Y_1^{-1} \left[ \frac{\alpha}{1-\alpha} \right] \end{aligned}$$

$$\mathcal{I}(Y, r) = T(Y, r) = e^t.$$

Which represent the particular solution of equation (5.3).

**6- Conclusion**

The Yang transform was employed to solve the transport and Poisson equations and to derive general formulas for them, future work will involve using the transform to solve certain systems with variable coefficients.

**References:**

- [1] Goodwine, "Engineering Differential Equations: Theory and Applications", Springer, New York, USA, (2010).
- [2] F.W. Dorr " the direct solution of the discrete Poisson equation on a rectangle " siamreview , Vol. 12, April (1970).

- [3] Hussein NA. How to use Temimi Transformation for solving LODE without using any initial Conditions. journal of kerbala university. 2009;7(1):124-8.
- [4] A N Albukhuttar ,Z D Ridha, H N Kadhim Applications of A Shehu Transform to the Heat and Transport Equations International Journal of Psychosocial Rehabilitation, Vol.24 (2), 2020.
- [5] N. Gabriel, " Ordinary Differential Equations ", Mathematics Department, Michigan State University, Vol 16, No 6 (2015), pp 123-143.
- [6] R.N. Bracewell, "The Fourier transform and its applications", McGraw-Hill, Boston, Mass, USA, 3rd edition, (2000).
- [7] L. Debnath and D. Bhatta, "Integral Transforms and Their Applications," Second Edition, Chapman & Hall, London,(2007).
- [8] ATM Elzaki and SM Ezaki, "Application of New Transform "Elzaki Transform" to Partial Differential Equations", Number 2011.
- [9] H.M. Ali and N.K. Athraa, "On Solutions of Differential Equations by using Laplace Transformation", Journal of Islamic University, Vol 3, No1(2010), pp 29-37.
- [10] G.K Watugala, "Sumudu transform - a new integral transform to solve differential equations and control engineering problems", Math. Engg. in Indust, vol. 6, 1998.
- [11] A. Siegman, "Quasi fast Hankel transform", Optics Letters, vol. 1, pp. 13-15, 1977.
- [12] L. Boyadjiev and Y. Luchko, "Mellin integral transform approach to analyze the multidimensional diffusion-wave equations", Chaos, Solitons& Fractals, vol. 102, pp. 127-134, 2017
- [13] Maha .S and Zaniab . Yang Transformation for Solving Ordinary Differential Equation Full Text Book of Rimar Congress of Pure and Applied Sciences.
- [14] X.J. Yang, "A new integral transform method for solving steady heat-transfer problem", Thermal Science, vol. 20, pp. 639-642, 2016.
- [15] S. Qureshi and P. Kumar, "Using Shehu Integral Transform to Solve Fractional Order Caputo Type Initial Value Problems", Vol.18, 2019.