



Science Forum (Journal of Pure and Applied Sciences)

Journal Homepage: <https://atbuscienceforum.com.ng/index.php/jpas>



Improving Active Radon Data Modeling Using a Hybrid Method that Combines Neville Interpolation with Graph method using the BFGS Algorithm

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Abstract

This study presents a new method for improving modeling based on two methods: Neville's method and Graph method, using a shrinkage fusion method with weights calculated using the BFGS algorithm. To evaluate the performance of the Hybrid method and the basic methods in modeling, data on active radioactive radon gas were used in a number of selected areas of Baghdad. The importance of the study lies in improving the accuracy of modeling radon data, which is a hazardous gas with a radioactive effect, as traditional numerical methods may not provide sufficient accuracy in all cases. The results showed that Hybrid method outperformed the two separate methods, achieving the lowest sum of square errors (0.00353) compared to Neville's method (0.001432) and the Graph method (0.036126). the analysis also confirmed that the optimal combination of the two methods using calculated weights reduces deviations and improves the reliability of the results, making this method ideal for practical applications in monitoring active radioactive pollutants.

Keywords

Radon Gas,
Neville Interpolation,
Numerical Optimization,
BFGS Algorithm,
Hybrid Method.



Introduction

Mathematical modeling is an essential tool for understanding phenomena and analyzing relationships between variables. It enables to represent real-world problems using equations, functions, or mathematical systems. It helps predict, explain, and make decisions based on logical analysis. The most important aspect of modeling is simplifying complex phenomena into manageable models. It also allows for testing hypothesis without the need for costly or dangerous experiments. It enables predications of future system behavior and facilitates communication between researchers from different disciplines using a unified mathematical language.

Numerical methods are a vital tool in modeling scientific and engineering data, especially when dealing with precis environmental measurements. Examples of these methods include Neville's method. This numerical method is used to calculate an approximate value of a function at a given point using interpolation. It relies on alterative squares to gradually construct the interpolation value from known points. It does not require a closed-form formula for the function and is accurate in estimation, especially with a small number of points. It's implemented in a symmetric iterative manner, making it suitable for computing. Examples of use this method include: generalized the Neville- Aitken algorithm to operate on Grassmann manifolds to approximate reduces models [1], reviewed the Neville method as a fundamental interpolation tool in a comparative analysis of data approximation methods [2], using Neville filters in the interpolation stage of a multi-stage spectral fusion algorithm [3], and applied the Neville method improve pulse amplitude measurement when taking [4].

Other medaling methods include the graphical method, which relies on representing relationships between variable using graphs. Point is plotted and then pattern (such as a straight line, curve, or a specific function) that describes the relationship

between the variables is attempted, it is easy to understand, even for non-experts, and reveals trends, outliers, and change points. It also helps in choosing the appropriate function type (linear, quadratic, logarithmic, etc.). examples of the use this method include: introducing a new model that combines flow models and graphical models [5], using it to analyze dependency between variables in extreme cases [6], and using graphical transformations in engineering design [7].

The main reason for adopting these methods to create a hybrid model is that the Neville method is known for its high accuracy in polynomial interpolation, making it suitable for data with rapid changes, and the graphical method provides flexibility in representing general trends. However, both methods suffer from limitations when applied alone; the Neville method may not capture fine details. Hence, the motivation to combine the two methods into hybrid model that achieves a balance between local accuracy and general flexibility, using BFGS algorithm to calculate optimal weights that minimize errors.

The study aims to develop a hybrid method that yields more accurate results than the individual methods compared to the proposed method and the basic methods in estimating theoretical values for experimental data and analyzing the mean squared errors and their sum to evaluate the performance of each method.

Methods

Neville's Method

It's an algorithm used in interpolation to calculate an approximate value of a function at given point using a given set point. This method relies on repeatedly constricting a graduate table of polynomial interpolations, starting with first-order interpolations and moving up to higher-order ones. Therefore, the steps for interpolation Neville's method are as follows [8]:

- Setting the points: a set of known points $(x_0, y_0), \dots, (x_n, y_n)$ is selected to be used for approximation.
- Creating a table consisting of $n + 1$ rows and columns.

- Interpolating Neville's method by putting $y_i = Q_{i,i}$ the $Q_{i,j}$ as shown in the equation below:

$$Q_{i,j} = \frac{(x - x_i)Q_{i+1,j-1} - (x - x_{i+j})Q_{i,j-1}}{x_{i+j} - x_i}, i = 0, 1, \dots, n-j, j = 1, \dots, n \quad (1)$$

- Return to $Q_{0,n}$ as the final fulfillment value.

2.1 Graph method

The graph theory is a branch of mathematics that studies relationships and interactions between different objects and uses graphs to visually represent these relationships, with applications extending to various fields such as computational science, networks, social relationships, referrals, transportation, and many others. The graph method focuses on the study objects known as vertices and edges that connect them. To build a mathematical model of the graph, the following equation:

$$f_{bi} = f_{b0} + \frac{f_{bi} - f_{b0}}{x_i - x_0} \quad (2)$$

where f_{bi} first approximate value, f_{b0} is the true value at initial step, and x_i, x_0 are the effect rate at steps i and initial respectively [9].

1. Hybrid Method

This method combines the Neville method and graphical method this algorithm begins with the following steps:

- Define the basic methods (Neville method and graphical method), which was shown in equations (1) and (2).
- Define the weights these are coefficients used to combine the two methods. These coefficients can be fixed or variable values depending on the nature of the results of each method.
- Formulate the equation for the hybrid method, which is given as follows:

$$Hybrid = \omega_1 Neville + \omega_2 Graph \quad (3)$$

where ω_1, ω_2 are the weights of the hybrid equation, and $Neville, Graph$ are the results of Neville method and graphical method respectively.

- Select the optimal weights ω_1 and ω_2 using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm, which is a machine learning algorithm formulated in following steps:

- Initialize the weights ω_1 and ω_2 using random values.
- Calculate the Mean square error (MSE).
- Update the weights using equation:

$$\omega_1 = \omega_{1,0} - \theta \frac{\partial MSE}{\partial \omega_1} \quad (4)$$

$$\omega_2 = \omega_{2,0} - \theta \frac{\partial MSE}{\partial \omega_2} \quad (5)$$

where θ is the learn rate, and $\omega_{1,0}$ and $\omega_{2,0}$ are weights at initial step (Randomly selected values).

- Repeat the steps to obtain the best weights.
- Apply the hybrid method.
 - Calculate the errors at each point for effect rate using the equation:

$$Error = |Exp - Theor| \quad (6)$$
 where $Exp, Theor$ are the Experimental and theoretical values respectively.
 - Results analysis, which focuses on analyzing the accuracy of the hybrid method in terms of speed and accuracy of results.

2. Application On Real Data

The data used in this paper represented the concentration of effective radon gas inside building in some areas of Baghdad Governorate using the nuclear trace detector (CR-39) and measured in units of Bq/m^3 obtained from [10].

The data modeling process begins, as presented in the previous section of the study, by calculating the experimental values for both the Neville and graphical method. The errors between the actual and experimental values are also calculated at each point, and the square of these errors is then calculated to determine the efficiency of each method and the bias relative to actual values. Tables 1 and 2 represent the results of Neville and graphical method respectively. Figures 1 to 4 show the results of Neville method and its squared errors, and graphical method and its squared errors respectively.

TABLE1. Results for Neville method with errors

No.	Effect rate (x)	Exp.	Theor. Neville method	Error	Square error
1	23.33	46.280000	46.280304	0.000304	0.000000092416
2	9.66	19.170000	19.169400	0.000600	0.000000360000
3	22.77	45.180000	45.180490	0.000490	0.000000240100
4	26.11	51.790000	51.790739	0.000739	0.000000546121
5	3.22	6.390000	6.391712	0.001712	0.000002930944
6	5.44	10.790000	10.789806	0.000194	0.000000037636
7	10.33	20.490000	20.487862	0.002138	0.000004571044
8	18.22	36.140000	36.139160	0.000840	0.000000705600
9	18	35.700000	35.701355	0.001355	0.000001836025
10	36.11	71.630000	71.628928	0.001072	0.000001149184
11	19.44	38.570000	38.570961	0.000961	0.000000923521
12	16.66	33.060000	33.060124	0.000124	0.000000015376
13	17.7	34.600000	34.601437	0.001437	0.000002064969
Sum				0.011966	0.000143185156

TABLE2. Results for Graph method with errors

No.	Effect rate (x)	Exp.	Theor. Graph method	Error	Square error
1	23.33	46.280000	46.281533	0.001533	0.000002350089
2	9.66	19.170000	19.165454	0.004546	0.000020666116
3	22.77	45.180000	45.170807	0.009193	0.000084511249
4	26.11	51.790000	51.795430	0.005430	0.000029484900
5	3.22	6.390000	6.388058	0.001942	0.000003771364
6	5.44	10.790000	10.792970	0.002970	0.000008820900
7	10.33	20.490000	20.494646	0.004646	0.000021585316
8	18.22	36.140000	36.145927	0.005927	0.000035129329
9	18	35.700000	35.709546	0.009546	0.000091125316
10	36.11	71.630000	71.629484	0.000516	0.000000266256
11	19.44	38.570000	38.565833	0.004167	0.000017363889
12	16.66	33.060000	33.051559	0.008441	0.000071250481
13	17.7	34.600000	34.598752	0.001248	0.000001557504
Sum				0.060105	0.003612611025

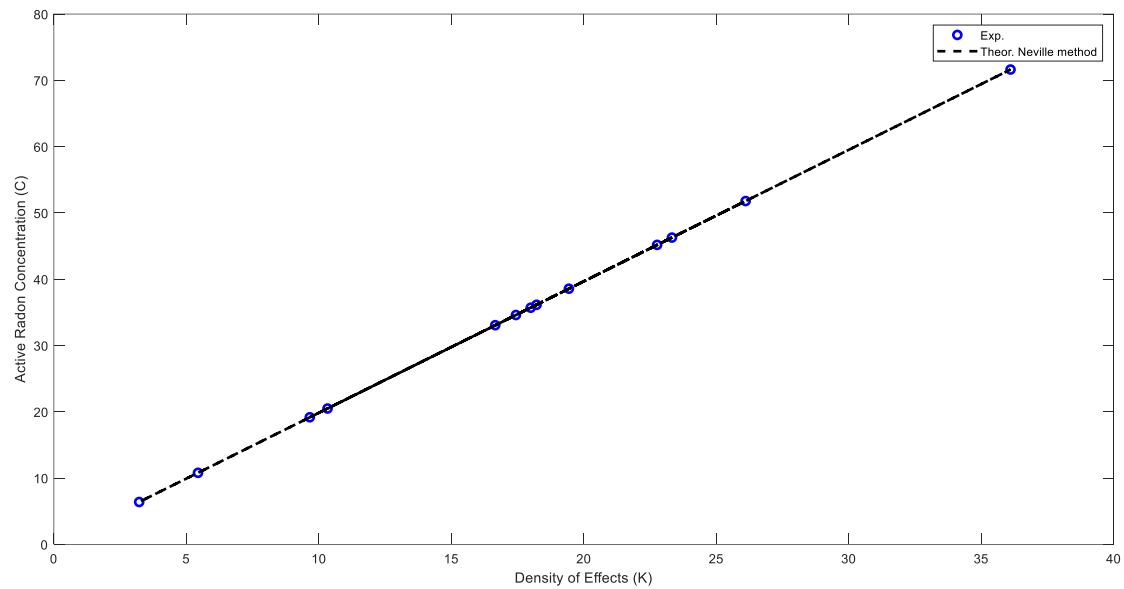


Figure 1. Results for Neville Method

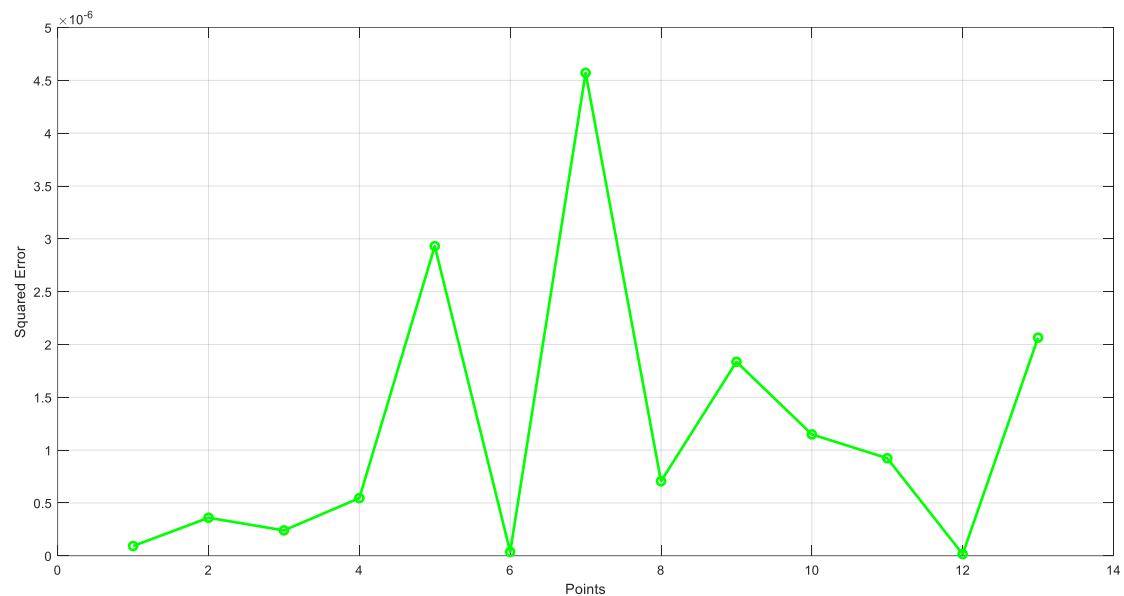


Figure 2. Squared Errors - Neville Method

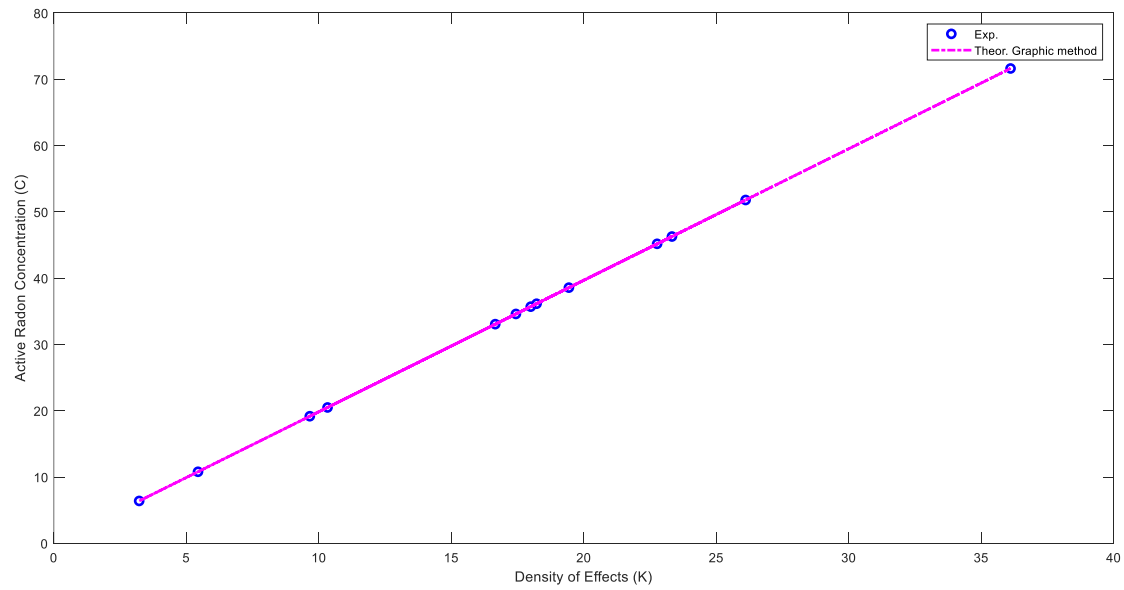


Figure 3. Results for Graph Method

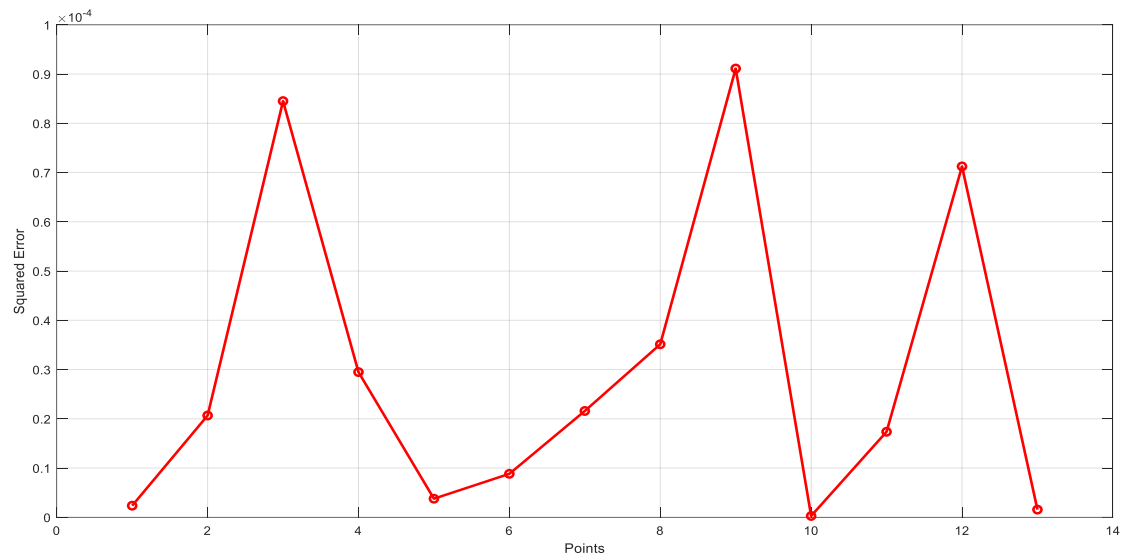


Figure4. Squared Errors - Graph Method

The weights at each point are now calculated using the BFGS algorithm. Table 3 presents the weights at each point method based on what was founded in tables 1 and 2, which confirms the importance of using BFGS algorithm to calculate weights based on the lowest error for each point, at was also verified that the sum of the weights equals 1.

TABLE3. Weights of the Neville interpolation and Graph methods for each point

No.	Nevil method	Graph method	Sum
1	0.834344	0.165656	1
2	0.883197	0.116803	1
3	0.949306	0.050694	1
4	0.880039	0.119961	1
5	0.531430	0.468570	1
6	0.938373	0.061627	1
7	0.684774	0.315226	1
8	0.875810	0.124190	1
9	0.875665	0.124335	1
10	0.325052	0.674948	1
11	0.812491	0.187509	1
12	0.985404	0.014596	1
13	0.464856	0.535144	1

After obtaining all parts of equation 6, it is used to obtain the results of the hybrid method, table 3 represent the results of hybrid method, Figures 5 and 6 show the results of hybrid method and its squared errors respectively.

Table 3. Results For Hybrid Method with Errors

No.	Effect rate (x)	Exp.	Theor. hybrid method	Error	Square error
1	23.33	46.280000	46.280507	0.000507	0.000000257049
2	9.66	19.170000	19.168939	0.001061	0.000001125721
3	22.77	45.180000	45.179999	0.000001	0.000000000001
4	26.11	51.790000	51.791302	0.001302	0.000001695204
5	3.22	6.390000	6.390000	0.000000	0.000000000000
6	5.44	10.790000	10.790001	0.000001	0.000000000001
7	10.33	20.490000	20.490000	0.000000	0.000000000000
8	18.22	36.140000	36.140001	0.000001	0.000000000001
9	18	35.700000	35.702373	0.002373	0.000005631129
10	36.11	71.630000	71.629303	0.000697	0.000000485809
11	19.44	38.570000	38.569999	0.000001	0.000000000001
12	16.66	33.060000	33.059999	0.000001	0.000000000001
13	17.7	34.600000	34.600000	0.000000	0.000000000000
Sum				0.005945	0.000035343025

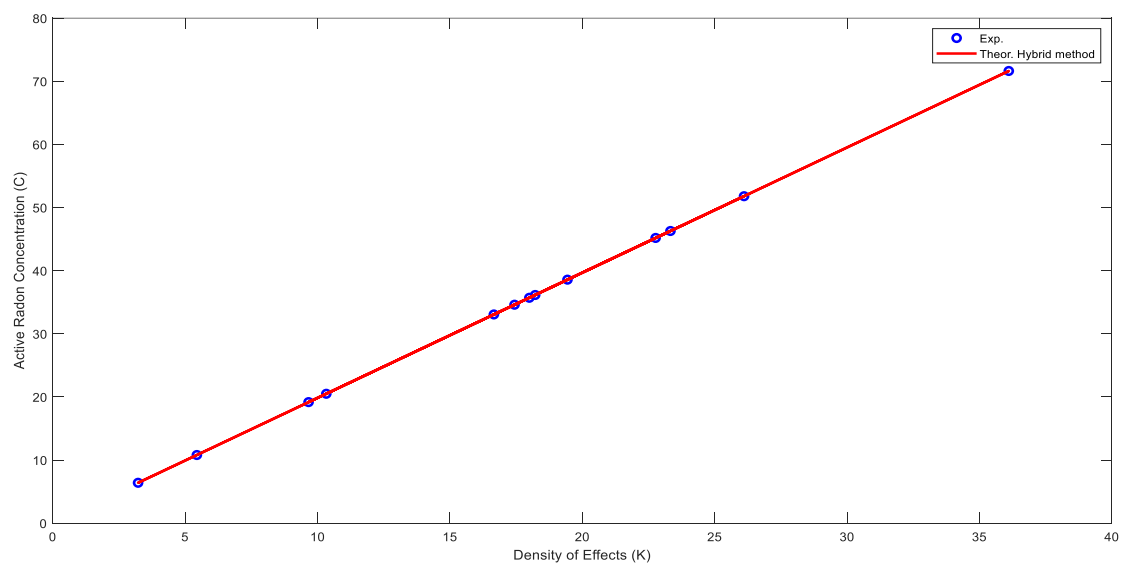


Figure 5. Results For Hybrid Method

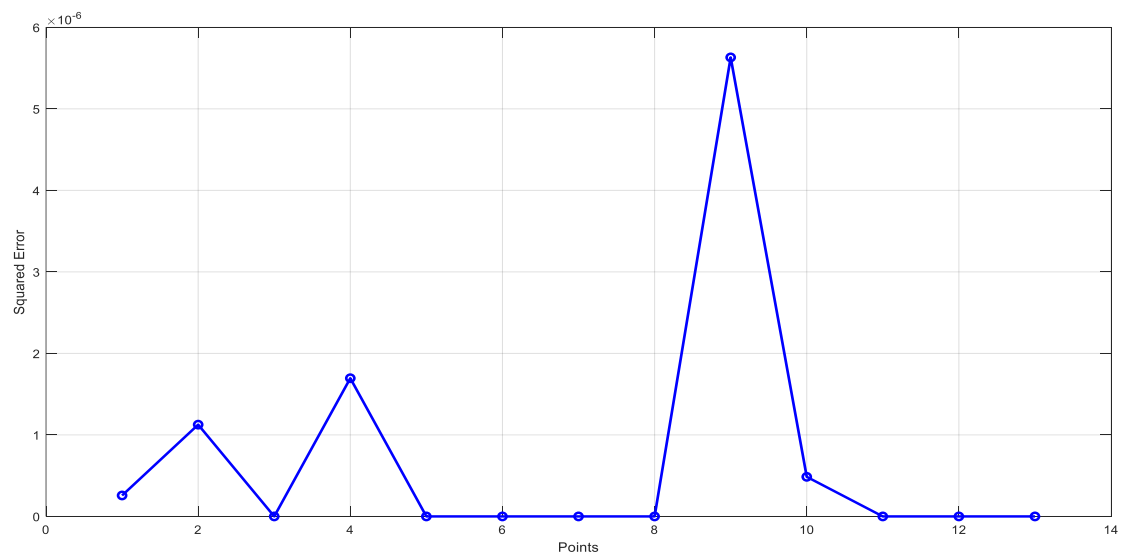


Figure 6. Squared Errors - Hybrid Method

3. Results and Comparison

Comparing the relative errors of each of the three methods, it is clear that the hybrid method achieved the lowest relative error rate compared to the other two methods. This indicates that combining the two methods using the BFGS algorithm to calculate the optimal weights succeeded in reducing deviations and improving the accuracy of the results. This is illustrated in figure 7, optimal weights (such as 0.834 for Neville and 0.166) for graph method in the first point) were optimized to ensure an optimal combination of the two methods and reduced the standard deviation of errors by up to 90% compared to the individual methods.

The errors in Neville method were very small but concentrated in specific points, indicating that the

method is accurate but may fail to capture fine details in some areas. The graph method showed greater variation in errors, with some high values, indicating that this method is less accurate in some points. The hybrid method achieved an excellent balance, with small consistent errors across all points, confirming the effectiveness of the combination of two methods, as seen in figure 8.

Figure 9 represent a comparison of the sum of squared errors of three methods. The results were as follows: Neville methods 0.000143185156, graph method 0.003612611025, and hybrid method 0.000035343025. The hybrid method achieved the lowest sum of squared errors, confirming its superiority in reducing deviations and improving the reliability of results compared to individual methods.

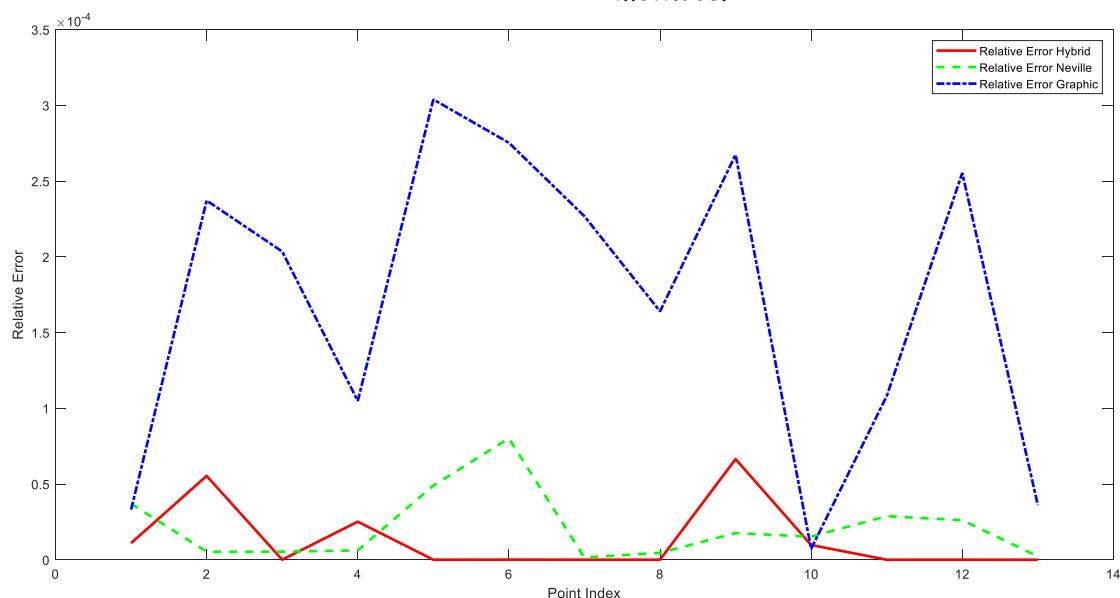


Figure 7. Relative Error Comparison

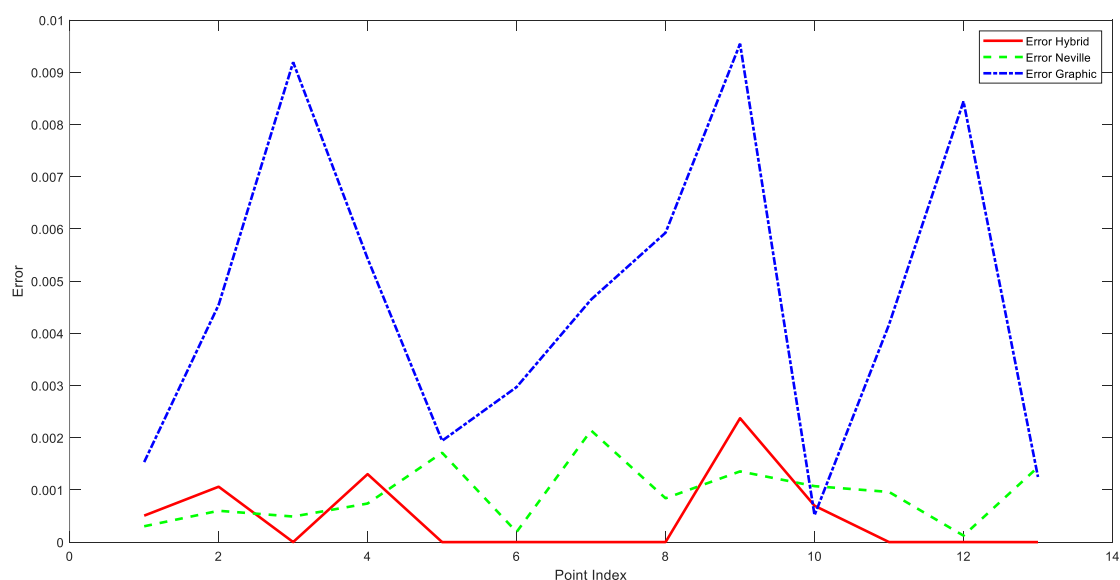


Figure 8. Errors for Each Method

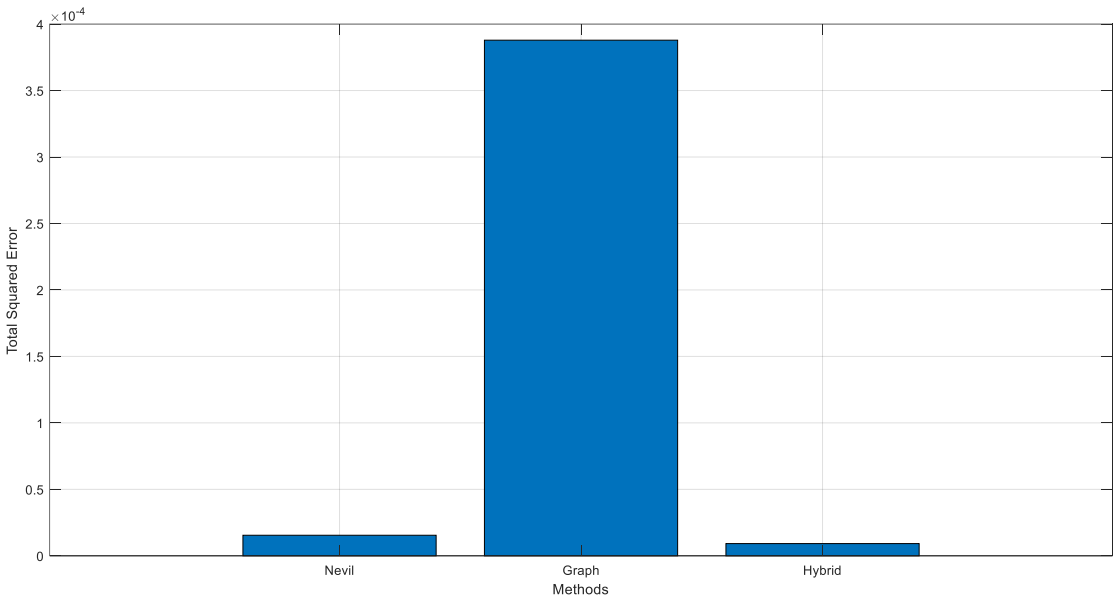


FIGURE9. Performance of Methods Based on Total Squared Errors

Conclusion

The study presents an effective hybrid method that improves the accuracy of modeling radioactive radon data, highlighting the importance of combining numerical and graph methods using machine learning techniques such as the BFGS algorithm. The hybrid method, which combines the Neville method and graph method using BFGS algorithm to calculate optimal weights, demonstrated clear superiority in the accuracy of the results, achieving the, lowest square error (0.000035343025) compared to individual methods. It successfully overcame the limitations of the other two methods, providing high accuracy in capturing local details (thanks to the Neville method) and flexibility in representing overall trends (thanks to the graphical method).

The BFGS algorithm played a crucial role in optimizing the weights used in the integration, which contributed to reducing errors and improving the efficiency of the hybrid model. This hybrid method can be used in monitoring radioactive pollutants such as radon gas, as it provides high

accuracy and reliability in results, making it ideal for practical applications in environmental and engineering fields. The results are consistent with the main objective of the study, which is to develop a hybrid model that provides more accurate results than traditional methods, enhancing its potential for use in analyzing complex data.

The study demonstrated that the intelligent combination of numerical methods using optimization techniques can achieve a leap in modeling accuracy, especially in sensitive fields such as radiation monitoring. The results are not limited to radon gas, but can be applied to any environmental or industrial dataset that requires a balance between analytical and visual representation. This research opens new avenues for improving hybrid numerical models in applied sciences.

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