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Cut-Points with Separations in Semi and Pre-Connected Topological Spaces

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Abstract

This paper presents a systematic comparison of cut-points in semi-connected and pre-connected topological spaces. Starting from semi-open, pre-open, and Njåstad alpha-open sets, we derive explicit formulas for semi-closure and pre-closure and prove the identity $\alpha O(X) = SO(X) \cap PO(X)$. We then establish a central structural result: with the Njåstad definition adopted here, alpha-connectedness is equivalent to ordinary connectedness. It follows that classical cut-points and alpha-cut-points coincide, so the distinct behavior of generalized cut-points occurs at the semi-open and pre-open levels. In a semi-connected space, every classical cut-point is a semi-cut-point; similarly, in a pre-connected space, every classical cut-point is a pre-cut-point. Neither implication reverses in general. We construct two explicit finite topological spaces demonstrating that semi-cut-points and pre-cut-points are independent, even when each ambient space is both semi-connected and pre-connected. To clarify the role of subspace topology, we also provide a detailed induced-subspace calculation on a three-point space. This example highlights a limitation of direct component-based extensions: semi-openness is not hereditary under arbitrary passage to subspaces. Consequently, a semi-component need not be contained in one side of a semi-separation unless an additional hypothesis is imposed. Together, these results distinguish unconditional transfer principles from claims that require further assumptions. The framework clarifies the relationships among classical, semi-, pre-, and alpha-cut-points, while situating the comparison within recent research on generalized openness, pretopological connectivity, and graph-associated topologies. It provides explicit finite models for testing proposed implications and emphasizes the need to verify heredity before importing component arguments into generalized settings through finite examples.

Keywords

Alpha-open sets;
Cut-points;
Pre-connectedness;
Pre-open sets;
Semi-connectedness;
Semi-open sets.



1. Introduction

Connectedness is one of the central global properties of a topological space, with standard background available in classical texts such as Kelley [5] and Willard [6]. A classical cut-point is a point whose deletion destroys connectedness, and the distribution of such points often reveals the internal structure of the space. Generalized open sets allow the same question to be asked with weaker notions of openness. Semi-open sets were introduced by Levine in 1963 [1], alpha-open sets by Njåstad in 1965 [2], and pre-open sets by Mashhour, Abd El-Monsef, and El-Deeb in 1982 [3]. These classes have generated corresponding notions of continuity, separation, compactness, and connectedness.

The semi-open and pre-open conditions differ only in the order of the interior and closure operators, but this difference is mathematically significant. A set A is semi-open when $A \subseteq Cl(Int(A))$, while it is pre-open when $A \subseteq Int(Cl(A))$. Neither family contains the other in general; the behavior of pre-open sets has been studied extensively, for example by Ganster [11]. Hence semi-connectedness and pre-connectedness need not agree, and the cut-points obtained from them need not agree either. The semi-connected and pre-connected notions adopted in this paper follow the conventions cited in [7] and [8], respectively.

Mahmoud and Yousif [9] studied cut-points and separations in alpha-connected topological spaces in a graph-motivated setting; related compatibility and edge-space constructions were developed by the same authors in [10]. Their work naturally raises the question of which statements remain valid when alpha-openness is replaced by semi-openness or pre-openness. A rigorous answer requires more than replacing one adjective by another. In particular, generalized openness may fail to behave hereditarily with respect to arbitrary subspaces, so arguments copied from ordinary connectedness can acquire hidden gaps.

Recent literature continues to develop these weak forms of openness and their associated structures. Connectedness based on generalizations of semi-open sets was investigated in 2022 [12]. Pre-open sets were used to construct generalized

regularity and separation conditions in 2024 [13]. Generalized primal topology was used in 2024 to compare semi-, pre-, and alpha-type open classes through implication results and counterexamples [14]. Graph-associated independent and incidence topologies with induced separation axioms appeared in 2025 [15], while multi-pretopological connectivity was developed in 2025 [16]. In 2026, supra delta-semi-open sets were used to define new separation and continuity structures [17].

The present paper investigates which cut-point and separation results persist when alpha-openness is replaced by semi-openness or pre-openness. Its main contributions are: (i) precise formulas for semi- and pre-closure; (ii) a proof that standard alpha-connectedness is equivalent to ordinary connectedness; (iii) the resulting equality of classical and alpha-cut-points; (iv) transfer theorems from classical cut-points to semi- and pre-cut-points; (v) explicit finite counterexamples establishing the independence of semi-cut-points and pre-cut-points; and (vi) a subspace example showing why component-type extensions require an additional hypothesis.

2. Preliminaries and Basic Definitions

Throughout, (X, τ) denotes a topological space and $A \subseteq X$. The symbols $Int(A)$ and $Cl(A)$ denote the ordinary interior and closure of A . For a subspace $Y \subseteq X$, the symbols Int_Y , Cl_Y , sCl_Y and pCl_Y refer to the interior, closure, semi-closure and pre-closure computed in the induced topology on Y . Generalized openness in Y is likewise always understood relative to the induced topology unless explicitly stated otherwise.

Definition 2.1. A subset A of X is semi-open if $A \subseteq Cl(Int(A))$ [1], pre-open if $A \subseteq Int(Cl(A))$ [3], and alpha-open if $A \subseteq Int(Cl(Int(A)))$ [2]. The corresponding families are denoted by $SO(X)$, $PO(X)$, and $\alpha O(X)$. A set is semi-closed (respectively, pre-closed) if its complement is semi-open (respectively, pre-open).

Definition 2.2. The semi-closure $sCl(A)$ is the intersection of all semi-closed sets containing A [4]. The pre-closure $pCl(A)$ is the intersection of all pre-closed sets containing A .

Definition 2.3. A space X is semi-connected if it is not the union of two disjoint nonempty semi-open subsets [7]. It is pre-connected if it is not the union of two disjoint nonempty pre-open subsets [8].

Alpha-connectedness and ordinary connectedness are defined analogously using alpha-open sets and open sets, respectively.

3. Foundational Relations, Cut-Points and Separation Results

Proposition 3.1. For every $A \subseteq X$,

$$sCl(A) = A \cup \text{Int}(Cl(A)), \quad pCl(A) = A \cup Cl(\text{Int}(A)).$$

Proof. A set F is semi-closed exactly when $\text{Int}(Cl(F)) \subseteq F$. Put $S = A \cup \text{Int}(Cl(A))$. Since $S \subseteq Cl(A)$, we have $\text{Int}(Cl(S)) \subseteq \text{Int}(Cl(A)) \subseteq S$, so S is semi-closed. If F is semi-closed and $A \subseteq F$, then $\text{Int}(Cl(A)) \subseteq \text{Int}(Cl(F)) \subseteq F$; hence $S \subseteq F$. Thus S is the smallest semi-closed superset of A . For pre-closure, F is pre-closed exactly when $Cl(\text{Int}(F)) \subseteq F$. Put $D = Cl(\text{Int}(A))$ and $P = A \cup D$. Let U be any open subset of P . As D is closed, $U \setminus D$ is open; moreover, $U \setminus D \subseteq A$, since $U \subseteq A \cup D$. Hence $U \setminus D \subseteq \text{Int}(A) \subseteq D$. But $(U \setminus D) \cap D = \emptyset$, so $U \setminus D = \emptyset$ and therefore $U \subseteq D$. Taking the union of all such U gives $\text{Int}(P) \subseteq D$. It follows that $Cl(\text{Int}(P)) \subseteq Cl(D) = D \subseteq P$, so P is pre-closed. Finally, whenever $A \subseteq F$ and F is pre-closed, $D = Cl(\text{Int}(A)) \subseteq Cl(\text{Int}(F)) \subseteq F$, whence $P \subseteq F$. Thus $P = pCl(A)$. $\square$

Proposition 3.2. For every $A \subseteq X$, A is alpha-open if and only if A is both semi-open and pre-open. Hence

$$\alpha O(X) = SO(X) \cap PO(X).$$

Proof. If A is alpha-open, then $A \subseteq \text{Int}(Cl(\text{Int}(A))) \subseteq Cl(\text{Int}(A))$, so A is semi-open. Also $\text{Int}(A) \subseteq A$ gives $Cl(\text{Int}(A)) \subseteq Cl(A)$, hence $A \subseteq \text{Int}(Cl(A))$; thus A is pre-open. Conversely, assume A is semi-open and pre-open. From $A \subseteq Cl(\text{Int}(A))$ we get $Cl(A) \subseteq Cl(\text{Int}(A))$. Therefore $A \subseteq \text{Int}(Cl(A)) \subseteq \text{Int}(Cl(\text{Int}(A)))$, so A is alpha-open. $\square$

Theorem 3.3. For the standard Njåstad alpha-open sets, a topological space is alpha-connected if and only if it is connected.

Proof. If X is disconnected, then $X = U \cup V$ for nonempty disjoint open sets U and V . Open sets are alpha-open, so X is alpha-disconnected. Conversely, suppose $X = A \cup B$ is an alpha-separation. By Proposition 3.2, A and B are both semi-open and pre-open. Since B is semi-open, $A = X \setminus B$ is semi-closed, so $\text{Int}(Cl(A)) \subseteq A$. Because A is pre-open, $A \subseteq \text{Int}(Cl(A))$. Therefore $A = \text{Int}(Cl(A))$, which is open. The same argument shows that B is open. Hence $\{A, B\}$ is an ordinary separation and X is disconnected. $\square$

Corollary 3.4. Every semi-connected space is connected and every pre-connected space is connected. Since connectedness and alpha-connectedness are equivalent, each is also alpha-connected. The converses fail in general, and semi-connectedness and pre-connectedness are mutually incomparable.

Proof. Every open set is both semi-open and pre-open. Thus an ordinary separation is simultaneously a semi- and a pre-separation. The counterexamples to the converses and to comparability are given in Section 4. $\square$

Definition 3.5. Two nonempty subsets A and B of a space Y are mutually semi-separated if $sCl_Y(A) \cap B = \emptyset$ and $A \cap sCl_Y(B) = \emptyset$. They are mutually pre-separated if $pCl_Y(A) \cap B = \emptyset$ and $A \cap pCl_Y(B) = \emptyset$. A semi-separation (respectively, pre-separation) of Y is a pair $\{A, B\}$ of nonempty mutually semi-separated (respectively, pre-separated) sets such that $Y = A \cup B$.

Proposition 3.6. For complementary nonempty subsets A, B of Y , $\{A, B\}$ is a semi-separation if and only if both A and B are semi-open in Y . The analogous equivalence holds for pre-separation and pre-open sets.

Proof. Assume first that A and B are semi-open and complementary. Then each is also semi-closed because its complement is semi-open; hence $sCl_Y(A) = A$ and $sCl_Y(B) = B$, so they are mutually semi-separated. Conversely, if they are mutually semi-separated and $Y = A \cup B$, then $sCl_Y(A) \subseteq A$ and $sCl_Y(B) \subseteq B$. Since $A \subseteq sCl_Y(A)$ and $B \subseteq sCl_Y(B)$, both are semi-closed. Their complements are therefore semi-open, so A and B are semi-open. The pre-case is identical. $\square$

Definition 3.7. Let X be semi-connected. A point $x \in X$ is a semi-cut-point if $X \setminus \{x\}$ is semi-disconnected. Let X be pre-connected. A point $x \in X$ is a pre-cut-point if $X \setminus \{x\}$ is pre-disconnected. Classical and alpha-cut-points are defined analogously, with connectedness and alpha-connectedness, respectively, required of the ambient space. Denote their sets by $C(X)$ and $C_\alpha(X)$; when X is semi-connected or pre-connected, denote the corresponding cut-point sets by $C_s(X)$ and $C_p(X)$, respectively.

Theorem 3.8. In every connected topological space, a point is a classical cut-point if and only if it is an alpha-cut-point. Thus the two cut-point sets coincide:

$$C(X) = C_\alpha(X).$$

Proof. For any $x \in X$, the deletion $Y = X \setminus \{x\}$ is a topological space in its induced topology. By Theorem 3.3, Y is connected if and only if it is alpha-connected. Therefore deleting x destroys ordinary connectedness exactly when it destroys alpha-connectedness. $\square$

Theorem 3.9. Let X be semi-connected. Every classical cut-point of X is a semi-cut-point. Let X be pre-connected. Every classical cut-point of X is a pre-cut-point. Consequently, if X is both semi-connected and pre-connected, then

$$C(X) = C_\alpha(X) \subseteq C_s(X) \cap C_p(X).$$

Proof. Let x be a classical cut-point and $Y = X \setminus \{x\}$. Then $Y = U \cup V$ for disjoint nonempty open sets U, V in Y . Since every open set is semi-open and pre-open, $\{U, V\}$ is both a semi-separation and a pre-separation of Y . Under the appropriate ambient connectedness assumption, x is therefore a semi-cut-point or a pre-cut-point, respectively. The equality $C(X) = C_\alpha(X)$ is Theorem 3.8. $\square$

Proposition 3.10. A space Y is semi-disconnected if and only if it contains a nonempty proper subset A that is both semi-open and semi-closed. Similarly, Y is pre-disconnected if and only if it has a nonempty proper pre-open and pre-closed subset.

Proof. If $Y = A \cup B$ is a semi-separation, then A and B are complementary semi-open sets; hence each is also semi-closed. Conversely, if A is nonempty,

proper, semi-open, and semi-closed, then $Y \setminus A$ is nonempty and semi-open, giving a semi-separation. The pre-case is identical. $\square$

Corollary 3.11. A point x of a semi-connected space is a semi-cut-point exactly when $X \setminus \{x\}$ contains a nontrivial semi-clopen subset (a nonempty proper subset that is both semi-open and semi-closed in $X \setminus \{x\}$). A point x of a pre-connected space is a pre-cut-point exactly when $X \setminus \{x\}$ contains a nontrivial pre-clopen subset (a nonempty proper subset that is both pre-open and pre-closed in $X \setminus \{x\}$).

4. Corrected Examples, Independence, and a Subspace Caution

Finite spaces are especially useful because every interior, closure, and generalized-open family can be computed exactly. The examples below also make clear which reverse implications fail.

Example 4.1 (three-point induced-subspace calculation). For a related three-point graph-associated example, see [9, Example 3.11], which uses a different topology. Here let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. The space X is connected. If a is removed, the induced topology on $Y = \{b, c\}$ is $\{\emptyset, \{b\}, Y\}$, because $\{a, b\} \cap Y = \{b\}$; in particular, $\{b\}$ cannot be omitted. The space Y remains connected because $\{c\}$ is not open. Removing b gives the topology $\{\emptyset, \{a\}, \{a, c\}\}$ on $\{a, c\}$, and removing c gives $\{\emptyset, \{a\}, \{a, b\}\}$ on $\{a, b\}$; both subspaces are connected. Each deletion subspace is also semi-connected and pre-connected. Thus X has no classical, alpha-, semi-, or pre-cut-points.

Example 4.2 (a pre-cut-point that is not a semi-cut-point). Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{a\}, X\}$. A direct calculation gives

$$SO(X) = PO(X) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}.$$

Every nonempty proper semi-open or pre-open set contains a . Therefore no pair of disjoint nonempty semi-open sets, and no pair of disjoint nonempty pre-open sets, can cover X . Thus X is both semi-connected and pre-connected. Delete a . The induced topology on $Y = \{b, c\}$ is indiscrete: $\tau_Y = \{\emptyset, Y\}$. In Y the only semi-open sets are $\emptyset$ and Y , so Y is semi-connected. In contrast, every subset of an indiscrete space is pre-open: for a nonempty proper $A \subset Y$, $Cl_Y(A) = Y$ and $Int_Y(Y) = Y$. Hence $\{b\}$ and $\{c\}$

form a pre-separation. Thus the deletion space is semi-connected but pre-disconnected, exhibiting the first direction of incomparability. Therefore a is a pre-cut-point but not a semi-cut-point. Since Y is connected, a is not a classical (and therefore not an alpha-) cut-point.

$$a \in C_p(X) \setminus C_s(X), \quad a \notin C(X) = C_\alpha(X).$$

Example 4.3 (a semi-cut-point that is not a pre-cut-point). Let $X = \{a, b, c, d\}$ and

$$\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}.$$

For this topology, the proper nonempty semi-open, pre-open, and alpha-open subsets of X are the same: $\{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}$, and $\{a, c, d\}$. Every such set contains a , so X is both semi-connected and pre-connected. Delete a and put $Y = \{b, c, d\}$. The induced topology is $\tau_Y = \{\emptyset, \{b\}, \{c\}, \{b, c\}, Y\}$. In Y the sets $\{c\}$ and $\{b, d\}$ are complementary semi-open sets; hence Y is semi-disconnected. The pre-open subsets of Y are $\emptyset, \{b\}, \{c\}, \{b, c\}, Y$, and none of the nonempty proper pre-open sets has a pre-open complement. Thus Y is pre-connected but semi-disconnected, exhibiting the opposite direction of incomparability. It is also connected. Consequently a is a semi-cut-point but neither a pre-cut-point nor a classical/alpha-cut-point.

$$a \in C_s(X) \setminus C_p(X), \quad a \notin C(X) = C_\alpha(X).$$

Theorem 4.4 (independence). Semi-cut-points and pre-cut-points are independent in general, even when the ambient space is simultaneously semi-connected and pre-connected.

Proof. Example 4.2 has a semi-connected but pre-disconnected deletion space and produces a pre-cut-point that is not a semi-cut-point. Conversely, Example 4.3 has a pre-connected but semi-disconnected deletion space and produces a semi-cut-point that is not a pre-cut-point. Both ambient spaces are semi-connected and pre-connected; therefore neither cut-point class contains the other in general. $\square$

Example 4.5 (why an unqualified semi-component statement can fail). Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. The semi-open family is

$$SO(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}.$$

Hence $U = \{a\}$ and $V = \{b, c\}$ form a semi-separation of X . Consider $K = \{a, c\}$. Its induced topology is $\{\emptyset, \{a\}, K\}$; in this two-point space the only semi-open subsets are $\emptyset, \{a\}, K$, so K is semi-connected. Moreover, K is maximal among semi-connected subsets of X (the other maximal semi-connected subset is $\{b, c\}$). Thus K is a semi-component in the natural maximal-subspace sense, yet K meets both U and V . Therefore the ordinary statement "a component lies entirely in one side of a separation" does not transfer automatically to semi-components.

Proposition 4.6 (corrected component form). Let $\{U, V\}$ be a semi-separation of X and let $K \subseteq X$ be semi-connected. If $U \cap K$ and $V \cap K$ are semi-open in the subspace K , then $K \subseteq U$ or $K \subseteq V$.

Proof. If K meets both U and V , then $K = (U \cap K) \cup (V \cap K)$ is a union of two disjoint nonempty semi-open subsets of K , contradicting semi-connectedness. $\square$

Remark 4.7. The extra hypothesis in Proposition 4.6 is essential because semi-openness is not hereditary to arbitrary subspaces. The same methodological warning applies to pre-open sets. Therefore deletion theorems should be formulated and proved in the induced subspace topology rather than by assuming that generalized openness automatically lifts between a subspace and its ambient space.

5. Comparison and Recent Context

The preceding results give a sharper comparison than a simple linear hierarchy. Alpha-openness sits at the intersection of semi-openness and pre-openness, but alpha-connectedness itself does not create a new connectedness class: it agrees with ordinary connectedness. The genuinely stronger connectedness requirements are semi-connectedness and pre-connectedness, and those two are incomparable.

Table 1. Comparison of connectedness and cut-point notions

Aspect	Classical / α	Semi	Pre
Open-class relation	$\tau \subseteq \alpha O(X)$; α -connected $\Leftrightarrow$ connected	$\alpha O(X) \subseteq SO(X)$	$\alpha O(X) \subseteq PO(X)$
Relation between generalized classes	$\alpha O(X) = SO(X) \cap PO(X)$	$SO(X)$ and $PO(X)$ are incomparable	$PO(X)$ and $SO(X)$ are incomparable
Connectedness	connected $\Leftrightarrow \alpha$ -connected	semi-connected $\Rightarrow$ connected	pre-connected $\Rightarrow$ connected
Cut-point relation (when defined)	$C(X) = C_\alpha(X)$ (X connected)	$C(X) \subseteq C_s(X)$ if X is semi-connected	$C(X) \subseteq C_p(X)$ if X is pre-connected
Reverse implication	—	Converse fails (Example 4.3)	Converse fails (Example 4.2)
Semi vs. pre cut-points	—	Incomparable with $C_p(X)$ (Theorem 4.4)	Incomparable with $C_s(X)$ (Theorem 4.4)

The recent literature supports the usefulness of this implication-and-counterexample approach. Mahmood and Naser [12] developed connectedness through generalized semi-open structures. Sadek [13] used pre-open sets to build generalized regularity and separation properties. Al-Saadi and Al-Malki [14] compared semi-, pre-, and alpha-type open classes in generalized primal spaces and emphasized non-reversible implications. Mousa, Hassan, and Jafar [15] constructed graph-associated bitopologies from incidence and independent topologies and studied their separation axioms. Osman et al. [16] developed pre-connected structures in a multi-pretopological setting, and Obaid and Sadek [17] introduced supra delta-semi-open structures with new separation notions in 2026. These developments also clarify the role of graph-associated topology in the present problem. A graph articulation vertex should not be identified automatically with a semi-cut-point or pre-cut-point: the answer depends on the chosen topology on the graph-related carrier. The results above can be applied after the corresponding semi-open and pre-open families have been determined. A natural continuation is to characterize graph-induced

topologies for which articulation vertices coincide with $C_s(X)$, $C_p(X)$, or their intersection.

6. Conclusion

The theory of cut-points in semi-connected and pre-connected spaces becomes clearer once the generalized open classes are separated from one another at the foundational level. The identity $\alpha O(X) = SO(X) \cap PO(X)$ is important, but the corresponding connectedness relation is stronger than a one-way implication: with standard alpha-open sets, alpha-connectedness is exactly ordinary connectedness. Hence classical and alpha-cut-points coincide.

Semi-connectedness and pre-connectedness are genuinely stronger than ordinary connectedness and are not comparable. Accordingly, their cut-point notions are also not comparable. The two finite counterexamples in Section 4 prove both directions of this independence while keeping the ambient space simultaneously semi-connected and pre-connected. The three-point example emphasizes correct induced-subspace calculations, and the semi-component counterexample shows that generalized

openness cannot be moved between arbitrary subspaces without justification.

The resulting framework is both more economical and more rigorous: classical and alpha-cut-points form one class, while semi- and pre-cut-points provide two independent refinements. Future work may investigate when these refinements coincide under additional separation or irresolvability hypotheses, their preservation under suitable irresolute mappings, and their behavior in explicitly defined graph-associated, generalized, primal, or supra topological structures.

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