https://doi.org/10.70882/cgfd1y49

SCIENCE FORUM (JOURNAL OF PURE AND APPLIED SCIENCES) 26, (4) (2026), 1197 - 1206

Science Forum (Journal of Pure and Applied Sciences)

Journal Homepage: https://atbuscienceforum.com.ng/index.php/jpas

A Three-dimensional predator-prey model with fear effect and food subsidy stability analysis

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Abstract

The Holling-II stage-structured predator-prey system in which predators are classified as immature or mature is a subject of this research. In the prey-predator model that incorporates intraspecific competitions, the goal is to investigate an effects of a prey's terror brought on by the dread of the mature predator. Pregnant female predators and prey shelters are present. The theoretical study demonstrated that the prey's anxiety may result in improved stability at a positive equilibrium point and included both local and global stability analysis at a system's three equilibrium points. Applying a minor perturbation and linearizing the set of formulas surrounding every equilibrium point, we examine the outcome to see whether the model systems revert to their initial state. We develop a three-dimensional prey-predator system that considers the effects of fear and time management and age structure in the predator population. By examining a stubby traits and global dynamics of predator-prey systems under a combined influence of these variables, a current study aims to build on these advancements. The accuracy of the theoretical results was confirmed by numerical analysis, which also confirmed the system's stability criteria close to the non-trivial equilibrium points.

Keywords
Pregnant Female Predators,
Fear Effect,
Stage-Structure,
Holling Type-II,
Food Subsidy.

EISSN: 3156-5190 (Online)
ISSN: 1119-4618 (Print)
JPAS
Faculty of Science, ATBU Bauchi

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1 INTRODUCTION

Predator–prey interactions remain one of a most fundamental and widely studied topics in mathematical ecology, as they provide a framework for exploring the persistence, extinction, and oscillatory behavior of populations. The classical Lotka–Volterra model introduced the idea of cyclic dynamics between predators and prey; however, real ecosystems are influenced by multiple ecological and behavioral mechanisms that cannot be captured by such simplified models. Consequently, recent studies have focused on incorporating additional biologically realistic factors such as a fear effect, stage-structured predator classes, supplemental food, time delays, and harvesting strategies, all of which substantially affect system stability and coexistence outcomes.

One important ecological factor is a fear effect. Even in an absence of actual predation, a mere presence of predators may cause prey species to adjust their behaviour, decrease in foraging activity and reproductive success . This indirect influence has a potential to greatly limit both increase in prey population and predator persistence .Wang and Zou (2017) claim that including fear responses to models of predator-prey alters a dynamics of a system, potentially making it more less stable or stable. Homeostasis according to a intensity of a terror effect. These results demonstrate that both direct consumption and indirect ecological pressure take part in predation .Another crucial extension of predator-prey models is a incorporation of stage-structured dynamics, where predators are divided into adult and juvenile classes. Juveniles often have different survival tactics or food requirements than adults. Li, Sun, and Liu (2023) developed the stage-structured predator-prey model with functional responses that capture this difference and exhibit more active behaviours than unstructured models.

Similarly, Mondal, Barman, and Alam (2021) analyzed a ways in which adult predators arouse fear in prey populations, showing that a combination of fear effects and stage structure can offer new insights into ecosystem management. Giving predators more food has also been shown to have a major impact on system results.By reducing predators' dependence on prey, more food sources can encourage coexistence and reduce a pressure of predation. However, excessive supplementation may lead to system instability by encouraging predator overpopulation.

By reducing predators' dependence on prey, more food sources can encourage coexistence and reduce a pressu of predation However, too much supplementation may lead to system instability by encouraging predator overpopulation.Kumar and Dubey (2021) looked at predator-prey systems with both more food and discrete delays, demonstrating how these factors impact splitting behaviour.In predator harvesting scenarios under toxicity, Gakkhar and Rani (2019) also investigated a dual role of additional changing system equilibriam and food in enhancing predator survival .

Furthermore, time delays brought on by behaviouralresponses, development, or gestation are inevitable in real ecosystems.Mo and Shao (2025) found that delays can result in complex oscillatory dynamics and changes in equilibrium in their study of delayed stage-structured predator-prey systems that included cooperative fear, contacts, and additional food. Furthermore, Wang, Shao, and Chai (2023) emphasised how complex branching patterns emerge in predator reproduction when fear impacts and

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gestation delays combine.These results demonstrate that in order to accurately represent ecological complexity, temporal lags must be integrated. Together, these studies demonstrate how delays, harvesting, stage structure, additional food and terror effects are some of a ways that ecological realism significantly changes predator-prey dynamics.Adding such systems to mathematical models not only advances conceptual knowledge but also provides practical insights for biodiversitypreservation, sustainable resource management , and pest management.By examining a stabibty traits and glohal dynamics of predator-prey systems under a combined influence of these variables, a current study aims to build on these advancements.

2 MODEL FORMULATION

To investigate a dynamic interactions between prey and predators, we develop a three-dimensional prey–predator system that considers the effects of fear and time management and age structure in the predator population. Let $x(t)$denote a prey density, $y(t)$ a young predator density, and $z(t)$ a mature predator density at time. The proposed system is expressed as follows:

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\frac{\beta xy}{1+hx}-fxz \\ \frac{dy}{dt} &= s+m+\frac{\beta xy}{1+hx}-cy-dy \\ \frac{dz}{dt} &= cy-dz \end{aligned} \qquad (2.1)$$

The dynamics of the prey are described by the first equation. With an inherent growth rate (r) and carrying capacity(k) , a population increase logistically. It decreases due to predation by juvenile predators following a Holling type-II functional response $\frac{\beta xy}{1+hx}$, and is further suppressed by a fear effect induced by adult predators, represented by the term fxz.

The second equation governs the juvenile predator dynamics. The juvenile population increases through a constant inflow of individuals, represented by the presence rate (s) and the recruitment from pregnant female predators (m). In addition, juveniles gain from predation on prey $\frac{\beta xy}{1+hx}$. The loss in juvenile density arises from natural mortality (dy) and maturation into adulthood at a transition rate (cy).

The third equation captures the dynamics of adult predators. Adults are recruited from the juvenile class at rate (cy), while they decline through natural mortality at rate(dz) .The system is considered in the biologically feasible region .This model captures the essential ecological processes of predator–prey interactions by combining logistic prey growth, Holling type-II predation, predator fear effects, regarding the predator populations age distribution. It provides the comprehensive framework to analyze a stability and long-term behavior of the system under various ecological conditions.

Table 1: parameters description

Parameter	Description
$x(t)$	Prey density
$y(t)$	young predator density
$z(t)$	mature predator density
r	Prey growth rate
K	Prey carrying capacity (internal competition)
f	Prey s fear level from an adult predator
$\boldsymbol{\beta}$	Predation rate the rate at which a predator attacks juvenile prey
h	Prey handling time
s	Predator juvenile presence rate
m	Presence rate of pregnant female predators
c	Transition rate juvenile to adult
d	The death rates of predator

It fulfils the prerequisites$x(0) \geq 0, y(0) \geq 0 and z(0) \geq 0.$

Theorem (2.1): The system's domain (2.1), R_+^3 is invariant in a positive sense.

Proof: the partial derivatives of the interaction functions that show up on the system right side(2.1) are continuous on R_+^3.In that sense, these functions are Lipschitizian and hence a solution $[\mathrm{x(t)}, \mathrm{y}(t), \mathrm{z}(t)]$ of system (2.1) with positive preliminary circumstances $[\mathrm{x}(0), \mathrm{y}(0), \mathrm{z}(0)]$ unique and is exists on $[0, \rho)$ where $0 < \rho \leq +\infty$.

When $x = 0 \rightarrow \dot{x} = 0$

When $y = 0$

$$\dot{y} = s + m + \frac{\beta x.0}{1 + hx} - (c + d).0 = s + m \geq 0$$

When $y = 0 \rightarrow \dot{y} = 0$

When $z = 0 \rightarrow \dot{z} = cy \geq 0$

Hence R_+^3 is favourable invariant for prey-predator system (2.1).

Theorem (2.2): All the solutions $(x(t), y(t), z(t))$ of the system (2.1), which initiate in $R_+^3 - \{0\}$ have a uniform boundary.

Demonstration: $T(t)$ is bounded if and only if $x(t), y(t), z(t)$ are bounded. Deriving $\dot{x}$ regard for time provides

$$\dot{x} = rx\left(1 - \frac{x}{K}\right) - \frac{\beta xy}{1 + hx} - fxz \leq rx(1 - \frac{x}{K}) < 0$$

$$0 \leq x(t) \leq X_{max} \quad , max = \{x(0), K\} \quad \forall t \geq 0$$

Let $w(t) = y(t) + z(t)$

$$\dot{w} = s + m + \frac{\beta xy}{1 + hx} - dy - dz$$

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$$\dot{w} = s + m + \frac{\beta xy}{1+hx} - dw$$
$$\dot{w} \le s + m + By - dw \le s + m + Bw - dw = s + m - (d - B)w$$
$$\dot{w} \le s + m - \mu w \qquad , \mu = d - B > 0$$
$$w(t) \le \max\left\{w(0), \frac{s+m}{\mu}\right\}, \quad \forall t \ge 0$$

If the condition $d > B$ is not met

$$T(t) = x(t) + y(t) + z(t)$$
$$\dot{T} = rx\left(1 - \frac{x}{K}\right) - fxz + s + m - d(y + z)$$
$$T(t) \le T_{max} \qquad , \forall t \ge 0$$

3 Existence of Equilibrium Points (EP):

In this part, we analysis a nonlinear ordinary differential system's local stability. We use the linearization technique to represent the dynamic behavior of a complicated system that is very nonlinear. After applying a minor perturbation and linearizing the set of formulas surrounding every equilibrium point, we examine the outcome to see whether the model systems revert to their initial state. There are four biologically significant equilibrium points (EP) in a system.

(i)Point of trivial equilibrium (PTE) $\mathcal{f}_1 = (0,0,0)$ exist if $s + m = 0$

(ii) Prey equilibrium point (PEP) $\mathcal{f}_2 = (x, 0,0)$ exist if $s + m = 0$ where $x = k$

(iii) Equilibrium point of juveniles and adults without prey $\mathcal{f}_3 = (0, \frac{s+m}{c+d}, \frac{c(s+m)}{d(c+d)})$.

(iv) Point of Positive equilibrium $\mathcal{f}_4 = (x^*, y^*, z^*)$.

Next we address the presence of interior equilibrium $\mathcal{f}_4$, where the following set of algebraic equations can be solved to determine a values of x^*, y^* and z^* .

From equation (3)

$$z^* = \frac{c}{d}y^*$$

From equation (2)

$$y^* = \frac{s+m}{(c+d) - \frac{\beta x^*}{1+hx^*}}$$

From equation (3)

$$r\left(1 - \frac{x^*}{K}\right) - \frac{\beta y^*}{1+hx^*} - fz^* = 0$$

Compensation for value z^*

$$r\left(1 - \frac{x^*}{K}\right) - \frac{\beta y^*}{1+hx^*} - f\frac{c}{d}y^* = 0$$
$$r\left(1 - \frac{x^*}{K}\right) = \left[\frac{\beta}{1+hx^*} + \frac{fc}{d}\right]\left[\frac{s+m}{(c+d) - \frac{\beta x^*}{1+hx^*}}\right]$$
$$r\left(1 - \frac{x^*}{k}\right)[(c+d)(1+hx^*) - \beta x^*] = (s+m)[\beta + \frac{fc}{d}(1+hx^*]$$
$$r[(c+d) + [(c+d)h - \beta]x^*] - \frac{r}{K}x^*[(c+d) + [(c+d)h - \beta]x^*] = (s+m)[\beta + \frac{fc}{d} + \frac{fch}{d}x^*]$$
$$Bx^{*2} + Dx^* + C = 0$$

Where

$$B = \frac{-r}{K}[(c+d)h - \beta]$$
$$D = [r[(c+d)h - \beta] - \frac{r}{K}(c+d) - \frac{fch}{d}(s+m)]$$
$$C = [r(c+d) - (s+m)\left(\beta + \frac{fc}{d}\right)]$$

4 Local Stability Analysis

We can learn a lot about the future of system (2.1) from the local stability study. So, the conditions of instability or stability for the previously mentioned will be covered in this section (EP) $\mathcal{f}_i, i = 1,2,3.4$. For that, linearizing System (2.1) about any (EP) (x, y, z), yields the following variation matrix:

$$J = \begin{bmatrix} \xi_{11} & \xi_{12} & \xi_{13} \\ \xi_{21} & \xi_{22} & \xi_{23} \\ \xi_{31} & \xi_{32} & \xi_{33} \end{bmatrix} \qquad (4.1)$$

where ,

$\xi_{11} = r\left(1 - \frac{2x}{K}\right) - \frac{\beta y}{(1+hx)^2} - fz,$

$\xi_{12} = \frac{-\beta x}{1+hx},$

$\xi_{13} = -fx$

$\xi_{21} = \frac{\beta y}{(1+hx)^2}$

$\xi_{22} = \frac{\beta x}{1+hx} - c - d$

$\xi_{23} = 0$

$\xi_{31} = 0$

$\xi_{32} = c$

$\xi_{33} = -d$

Then the following formula is used to determine the eigenvalues that identify each equilibrium point's local stability:

Theorem (4.1): The system (2.1) around $\mathcal{f}_1 = (0,0,0)$ is always saddle point that is unstable.

Demonstration: A variational matrix (4.1) at (PTE) is written as

$$J_{\mathcal{f}_1} = \begin{bmatrix} r & 0 & 0 \\ 0 & -(c+d) & 0 \\ 0 & c & -d \end{bmatrix} \qquad (4.2)$$

$\lambda_1 = r > 0$, $\lambda_2 = -(c+d) < 0$, $\lambda_3 = -d < 0$

$\mathcal{f}_1$ is always unstable point

Theorem (4.2): A system (2.1) around $\mathcal{f}_2 = (k, 0,0)$ is always unstable saddle point.

Proof: A variational matrix (4.1) at (PEP) is written as

$$J_{\mathcal{f}_2} = \begin{bmatrix} -r & \frac{-\beta K}{1+hK} & -fK \\ 0 & \frac{\beta K}{1+hK} - (c+d) & 0 \\ 0 & c & -d \end{bmatrix} \qquad (4.3)$$

$$\lambda_1 = -r < 0, \quad \lambda_2 = \frac{\beta K}{1+hK} - (c+d)$$
$$, \lambda_3 = -d < 0$$

Locally stable if $\frac{\beta K}{1+hK} < (c+d)$ and unstable if $\frac{\beta K}{1+hK} > (c+d)$.

Theorem (4.3): The system (2.1) around $\mathcal{f}_3 = (0, \frac{s+m}{c+d}, \frac{c(s+m)}{d(c+d)})$ is always saddle point that is unstable.

Proof: A variational matrix (4.1) at $\mathcal{f}_3$ is written as:

$$J_{\mathcal{f}_3} = \begin{bmatrix} r - \beta y_0 - f z_0 & 0 & 0 \\ \beta y_0 & -(c+d) & 0 \\ 0 & c & -d \end{bmatrix} \qquad (4.4)$$

$\lambda_1 = z_0 f - y_0 \beta - r$, $if\ z_0 f + y_0 \beta < r \to 0 < \lambda_1$

$$\lambda_2 = -(d+c) < 0$$

$$\lambda_3 = -d < 0$$

$\not{f}_3$ locally stable if $r < \beta\left(\frac{s+m}{c+d}\right) + f(\frac{c(s+m)}{d(c+d)})$ and unstable if $z_0 f + y_0\beta > r$.

Theorem (4.4) The system (2.1) around $\not{f}_4 = (x^*, y^*, z^*)$ is locally asymptotically stable provided that a subsequent condition is met:

$\phi_1 > 0, \phi_3 > 0$, and $\Delta = \phi_1\phi_2 - \phi_3 > 0$

Proof: A variational matrix of system (4.1) around $\not{f}_4$, is as follows:

$$J_{\not{f}_4} = \begin{bmatrix} r\left(1 - \frac{2x^*}{K}\right) - \frac{\beta y^*}{(1+hx^*)^2} - fz^* & \frac{-\beta x^*}{1+hx^*} & -fx^* \\ \frac{\beta y^*}{(1+hx^*)^2} & \frac{\beta x^*}{1+hx^*} - (c+d) & 0 \\ 0 & c & -d \end{bmatrix} \qquad (4.5)$$

A variational matrix of a system (2.1) around $\not{f}_4$ is given by

$$J_{\not{f}_4} = (\partial_{ij})_{3\times 3}$$

where, $\partial_{ij} = \xi_{ij}$ at (x^*, y^*, z^*) for all $i, j = 1,2,3$.

Evaluating the characteristic equation of $J_{\not{f}_4}$, gives

$\lambda^3 + \phi_1\lambda^2 + \phi_2\lambda + \phi_3 = 0$

where

$$\phi_1 = -(\partial_{11} + \partial_{22} + \partial_{33})$$
$$\phi_2 = \partial_{11}\partial_{22} + \partial_{11}\partial_{33} + \partial_{22}\partial_{33} - \partial_{23}\partial_{32}$$
$$\phi_3 = \partial_{11}\partial_{23}\partial_{32} - \partial_{11}\partial_{22}\partial_{33} - \partial_{13}\partial_{21}\partial_{32}$$

Routh-Hurwitz conditions, for eq. ($\lambda^3 + \phi_1\lambda^2 + \phi_2\lambda + \phi_3 = 0$) have all roots with negative real part, are: $\phi_1 > 0, \phi_3 > 0$, and $\Delta = \phi_1\phi_2 - \phi_3 > 0$. Therefore, $\not{f}_4$ is locally asymptotically stable, if the conditions ($\phi_1 > 0, \phi_3 > 0$, and $\Delta = \phi_1\phi_2 - \phi_3 > 0$) hold. ■

5 An Analysis of Global Stability (AGS)

This section examines a global stability analysis using a Lyapunov approach for the (EP) that are (LAS) of system (2.1).

Theorem (5.1): A (AEP) $\not{f}_4 =$ of system (2.1) is global asymptotical stable whenever it's local asymptotically stable and the following condition holds

$\frac{dv_1}{dt} \leq 0\ \forall(x, y, z) > 0\ and\ \frac{dv_1}{dt} = 0 \leftrightarrow (x, y, z) = (x^*, y^*, z^*)$

Proof: examine the following function

$$v_1 = \left(x - x^* - x^* ln\frac{x}{x^*}\right) + \left(y - y^* - y^* ln\frac{y}{y^*}\right) + (z - z^* - z^* ln\frac{z}{z^*})$$

It is clearly to see: $v_1(x, y, z) \in c^1(R_+^3, R), v_1(\not{f}_4) = 0$, and $v_1(x, y, z) > 0;\ \forall(x, y, z) \neq \not{f}_4$. After decomposing v_1 in relation to time, some algerbraic dealing is provided:

$$\frac{dv_1}{dt} = (x - x^*)\left[\frac{-r}{k}(x - x^*) - \left(\frac{\beta y}{1+hx} - \frac{\beta y^*}{1+hx^*}\right) - f(z - z^*)\right] + (y - y^*)\left[(s+m)\left(\frac{1}{y} - \frac{1}{y^*}\right) + \left(\frac{\beta x}{1+hx} - \frac{\beta x^*}{1+hx^*}\right)\right]$$
$$+ (z - z^*)[c(\frac{y}{z} - \frac{y^*}{z^*}]$$
$$\frac{dv_1}{dt} = \frac{-r}{k}(x - x^*)^2 - \frac{\beta}{(1+hx)(1+hx^*)}(xy - x^*y^*)^2 - f(xz - x^*z^*)^2 - (c+d)(y - y^*)^2 - d(z - z^*)^2$$

So, $\frac{dv_1}{dt} \leq 0$, and hence v_1 is a Lyapunov function. Therefore, $\not{f}_4$ is globally asymptotically stable (GAS) in R_+^3.

■

6 Numerical Simulation

Numerical simulations are performed in MATLAB to visualize a dynamic of a prey and predator populations. While phase portraits depict the interactions between species, time series graphs display population densities across time. The analytical findings for both local and global stability are validated by these simulations. The model was numerically simulated (using Python) to get the figures below.

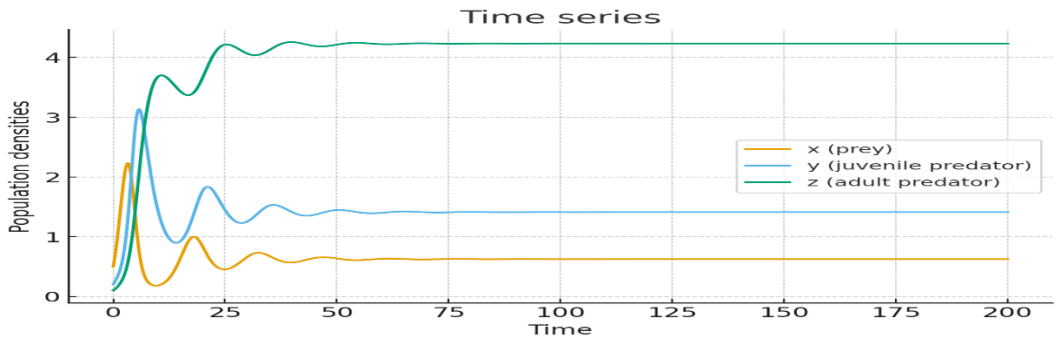


Figure 1: Time series of (x, y, z).

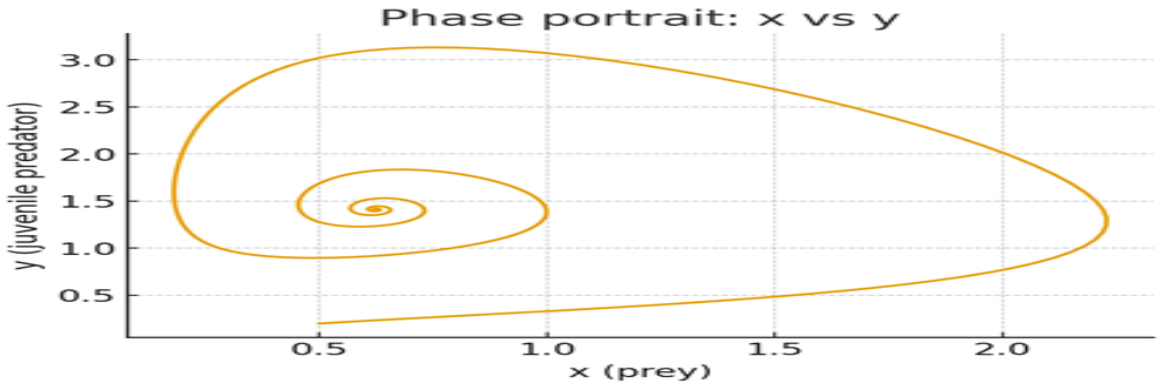


Figure 2: Portrait phase [x vs. y].

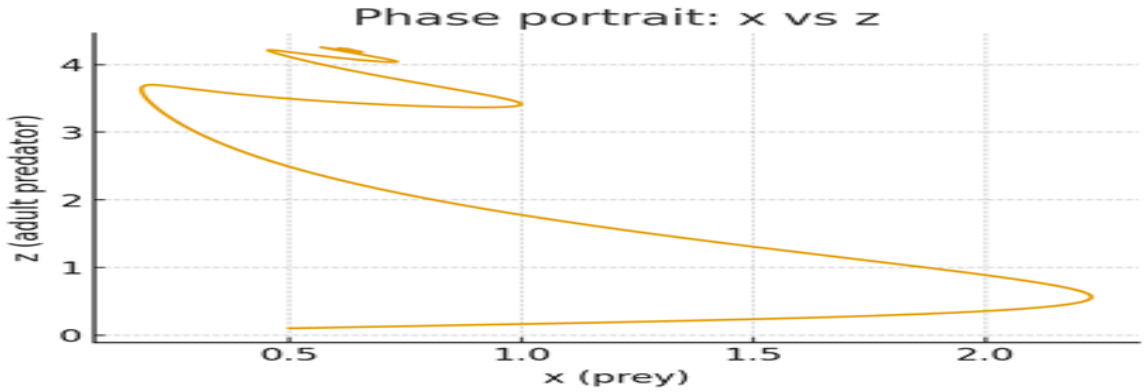


Figure 3: Portrait phase [x vs. z]

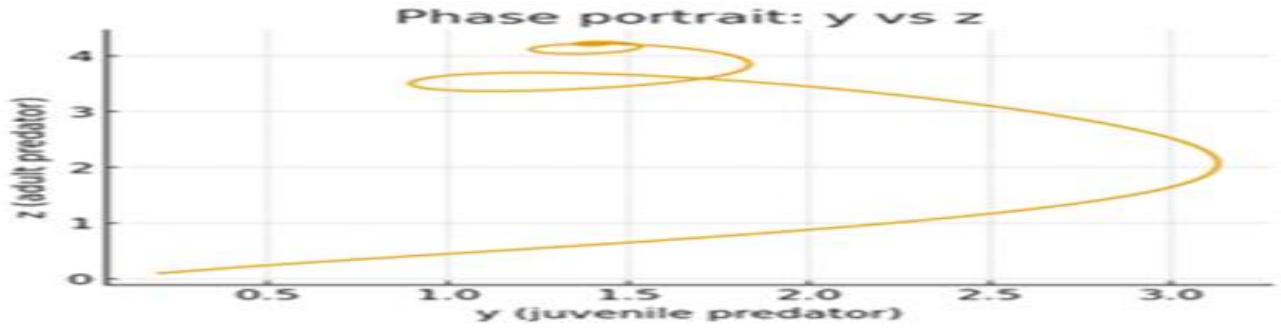


Figure4: portrait in phase [y vs. z]

7 Conclusions:

This study examines the existence of a pregnant female predator effect using numerical examination of the predator-prey system with a phase-structured hunter and fear. Investigations are conducted into the equilibrium points, local stability, and global stability. MATLAB simulations validate theoretical findings, which also show how fear and stage structure significantly affect population dynamics.

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