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The Characteristic Equation Method Solution for Partial Difference Equations

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Abstract

This study examines the role of the characteristic equation in solving linear partial difference equations. The question is whether this equation can provide a direct solution when differences occur with respect to multiple independent variables. The analysis indicates that linear partial difference equations cannot generally be solved by applying the characteristic equation in their original form. The proposed solution procedure first converts the partial difference equation into a linear difference equation by means of a suitable transformation method. Once this conversion has been completed, the resulting linear difference equation can be addressed using its characteristic equation. The characteristic equation thus remains relevant, but its use depends on prior transformation of the governing equation. This distinction establishes the relationship between the original partial difference equation and the subsequent solution process: transformation is a necessary preliminary step, while the characteristic equation is applied to the transformed equation. The study concludes that a direct solution of a linear partial difference equation cannot be obtained through the characteristic equation alone. Instead, solution is achieved indirectly, after conversion to a linear difference equation using one of the available transformation methods. The findings emphasize the importance of selecting an appropriate transformation before applying the characteristic-equation approach.

Keywords

Difference Equations,
Partial Linear Difference Equations,
Characteristic Equation,
Transformation Methods.

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INTRODUCTION

The characteristic equation is one of the most fundamental inclusions in the subject matter of mathematics and engineering at large. More specifically, this concept relates to linear algebra and differential equations. Its core purpose can be summarized as the process to factorize or break down a complex system (matrix, differential equation, or difference equation) into simple numeric values that reflect the behavior of the system. The characteristic equation of linear difference equations represents the basic means for solving equations that describe discrete systems. It is the analog of differential equations working with time sequences rather than derivatives. An equation of this type finds wide application in the analysis of algorithms, in financial mathematics (for example, in the theory of compound interest), in signal theory, and in digital systems.

These discrete differential equation counterparts show up whenever the independent variable takes on only discrete Values (Jasim, 2021, pp. 193-202) (Jasim A. S., 2025, pp. 146-152) (Roy, 2011, pp. 11-15) (Jensen, 2011, p. 44) (Kanasewich, 1981, p. 89) (Zadeh, 1952, pp. 225-234) (Leondes, 1996, p. 157) (Brand, 1955, pp. 489-492) (Elaydi, 2005, p. 176) (Bayram, 2016, pp. 2974-2982) (C. Liang, 2018, pp. 358-374) (M. Pospíšil, 2017, pp. 180-194) (Pospíšil, 2017, p. 482) (Stević, 2006, pp. 909-918)indicate that difference equations have applicability in problems arising in real situations and in different branches of science.

These may involve population dynamics, genetics, behavioral economics, sociology, combinatorial analysis, stochastic time series, radiation quanta, geometry, number theory, and electrical networks. Partial difference equations are those equations that contain functions of two or more independent variables. (R. Lynch, 1964, pp. 165-186) (W. Liang, 2008, pp. 717-733) (Mickens, 2015, pp. 289-296) (Zhang, 2018, p. 1950140)

This paper deals with the resolution of linear partial difference equations by means of the characteristic equation. It is divided into three sections: the first section gives the introduction, the second section discusses the characteristic equation, and the third section explains the method of transforming linear partial difference equations into linear difference equations.

2- Characteristic equation

If we have a homogeneous linear difference equation with constant coefficients of order k, it is written as follows:

$$a_k y_{n+k} + a_{k-1} y_{n+k-1} + \cdots + a_1 y_{n+1} + a_0 y_n = 0$$

To find the solution, we assume that the solution takes the form of a geometric sequence $y_n = r^n$ (where r≠ 0). Substituting this assumption into the equation and dividing all sides by r^n , we obtain the characteristic equation:

$$a_k r^k + a_{k-1} r^{k-1} + \cdots + a_1 r + a_0 = 0$$

Importance in control engineering and digital systems

In difference equations (discrete systems), the roots of the characteristic equation determine the stability of the system based on their position relative to the unit circle in the complex plane:

- Stable: If all roots lie within the unit circle, i.e., the absolute value of each root is ($|r| < 1$). In this case, the system tends towards zero over time.
- Unstable: If at least one root lies outside the unity circle ($|r| > 1$). Here, the values inflate to infinity.
- On the margin of stability (Marginally Stable): If the roots lie exactly on the

circumference of the unit circle (|r| = 1).

3. **Converting Partial Linear Equations of Difference into Ordinary Equations for Linear Differences** : (Liz, 2011, pp. 1471-1486) (D. T. Tollu, 2013, pp. 1-7) (S. Stević, 2016, pp. 147-165) (D. T. Tollu Y. Y., 2018, pp. 1765-1778) (Stević S. , 2018, p. 151) (Jasim E. A., 2021, pp. 51-56)

There are two methods of transferring:

3.1 The separation-of-variables method (Mickens, 2015, p. 142):

This method is used to get exceptional answers for straight halfway distinction conditions. To convert the partial difference equations to the linear difference equations, the general form for linear partial difference equations is:

$$\varphi(E_1, E_2, q, t)S(q, t) = 0, \quad (1_a)$$

where φ is a polynomial capability of E_1 and E_2.

The essential guideline of the technique is to accept that $S(q,t)$has the design is:

$$S(q, t) = v_i u_j, \quad (1_b)$$

where v_i and u_j are elements of q and t. Further, accept that when condition (1-b) is substituted into equation (1-a), an equation of the type is acquired:

$$\frac{f_1(E_1, q)v_q}{f_2(E_1, q)v_q} = \frac{g_1(E_2, t)u_t}{g_2(E_2, t)u_t} \quad (1_c)$$

Under these circumstances (v_q and u_t) the accompanying conventional contrast conditions:

$$f_1(E_1, q)v_q = \alpha f_2(E_1, q)v_q, \quad (1_d)$$

$$g_1(E_2, t)u_t = \alpha g_2(E_2, t)u_t,$$

where α is an arbitrary constant.

3.2 The Laplace's method (Brand, 1955, pp. 489-492)

Clarification on "Laplace's Method" for Partial Difference EquationsIt should be clarified that no discrete Laplace transform is intended by the term "Laplace's method" in this context; rather, the technique follows the classical approach outlined by Brand (1955) for solving linear difference equations. Change of Variables and Reduction Technique

The reduction is achieved by introducing a similarity-type change of variables based on the translation-invariant combination of the discrete arguments, namely:

$$Z = q - t$$

By substituting this invariant difference into the partial difference equation with constant coefficients, the multi-variable relation is reduced to a standard ordinary linear difference equation in terms of a single composite index.

Assumptions for the Arbitrary-Function Solution

The general solution involving arbitrary functions is valid under the following assumptions:

1. Constant Coefficients: The coefficients of the partial difference equation must be constants (or functions strictly dependent on the invariant combination).
2. Linearity and Homogeneity: The equation must be a linear homogeneous partial difference equation.
3. Shift Invariance: The arguments of the dependent variables must appear in combinations such that their differences remain constant, allowing the general solution to be expressed as a linear combination of exponential terms corresponding to the characteristic roots, multiplied by arbitrary functions of the invariant argument $(e.g., F(q - t))$.

This technique can be utilized when the total or distinctions of the contentions of all the $S(q + r, t + s)$ that show up in a halfway distinction condition are consistent, for

instance, think about the accompanying contrast condition with steady coefficients:

$$B_0S(q,t) + B_1S(q-1,t-1) + \cdots + B_nS(q-n,t-n) = X(q,t), \quad (2_a)$$

where $X(q,t)$ is a predefined function. Note that the distinction of the contentions $q-t$, is equivalent to a consistent. Presently, assuming we set $q-t=g$ also, put;

$$S(q,t) = S(q,q-g) = X_q, \quad (2_b)$$

Then equation (2-a) becomes;

$$B_0X_q + B_1X_{q-1} + \cdots + B_nX_{q-n} = X(q,q-g). \quad (2_c)$$

The arrangement of the last condition can be communicated as;

$$X_q = C_1\emptyset_1(q) + C_2\emptyset_2(q) + \cdots + C_n\emptyset_n(q) + P(q,g), \quad (2_d)$$

where the C_r is arbitrary constants; $\emptyset_r(q)$ for $r=1,2,3,\ldots,n,$ is a set of linerrly independent solution; and $P(q,g)$ is a particular be another steady. We can recuperate the answer for the first halfway contrast condition by supplanting q in conditions (2-d) by $q-t$ and arbitrary functions of $q-t$.

Doing the above gives;

$$S(q,t) = F_1(q-t)\emptyset_1(q) + F_2(q-t)\emptyset_2(q) + \cdots + F_n(q-t)\emptyset_n(q) + P(q,q-t), (2_F)$$

In addition, partial difference equations with coefficients that are functions of q and t can be solved using the aforementioned method.

Application:

Example 1: find the solution of the partial difference equation by Characteristic equation

$$S(q+1,t) = S(q,t+1)$$

Solution:

To solve this type of equation, we first transform the partial difference equation into a one-variable linear difference recurrence relation, and then apply its characteristic equation.

$$S(q+1,t) = S(q,t+1) \quad (E_1_a)$$

We will use the separation-of- variables method.

Let

$$S(q,t) = K_qY_t \quad (E_2_b)$$

By substituting Equation (E.2.b) into equation (E.1.a) yields;

$$K_{q+1}Y_t = K_qY_{t+1}$$

Or

$$\frac{K_{q+1}}{K_q} = \frac{Y_{t+1}}{Y_t} = \alpha, \quad (E_3_c)$$

Where α is arbitrary constant.

$$\frac{K_{q+1}}{K_q} = \alpha$$

$$K_{q+1} = \alpha K_q$$

$$K_{q+1} - \alpha K_q = 0, \quad (E_4_d)$$

Or

$$\frac{Y_{t+1}}{Y_t} = \alpha$$

$$Y_{t+1} - \alpha Y_t = 0, \quad (E_5_e)$$

the equations (E.4.d)and (E.5.e)are linear difference equations[6]

We will take Characteristic equation of equations (E_4_d)

$$K_{q+1} - \alpha K_q = 0$$

$$r^{q+1} - \alpha r^q = 0$$

$$r - \alpha = 0$$

$$r = \alpha$$

$$K_q = A\alpha^q$$

We will take Characteristic equation of equations (E_5_e)

$$Y_{t+1} - \alpha Y_t = 0$$

$$r^{t+1} - \alpha r^t = 0$$

$r - \alpha = 0$

$r = \alpha$

$Y_t = B\alpha^t$

Then $S(q,t) = K_q Y_t$

$= A\alpha^q B\alpha^t$

$= \boldsymbol{AB\ \alpha^{q+t}}$

Where A,B are arbitrary constants.

Summing over α allows us to obtain the general solution

$S(q,t) = ABf(q+t).$

Where f is an arbitrary function of $q+t$.

Example 2: find the solution of the partial difference equation by Characteristic equation

$S(q+1,t) - 3S(q,t+1) = 0$

Solution:

To solve this type of equation, we first transform the partial difference equation into a one-variable linear difference recurrence relation, and then apply its characteristic equation.

$$S(q+1,t) = 3S(q,t+1) \qquad (E_1_a)$$

We will use the separation-of- variables method.

Let

$$S(q,t) = K_q Y_t \qquad (E_2_b)$$

By substituting Equation (E.2.b) into equation (E.1.a) yields;

$K_{q+1} Y_t = 3K_q Y_{t+1}$

Or

$$\frac{K_{q+1}}{3K_q} = \frac{Y_{t+1}}{Y_t} = \alpha, \qquad (E_3_c)$$

Where α is arbitrary constant.

$\frac{K_{q+1}}{3K_q} = \alpha$

$K_{q+1} = 3\alpha K_q$

$$K_{q+1} - 3\alpha K_q = 0, \qquad (E_4_d)$$

Or

$\frac{Y_{t+1}}{Y_t} = \alpha$

$$Y_{t+1} - \alpha Y_t = 0, \qquad (E_5_e)$$

the equations (E.4.d)and (E.5.e)are linear difference equations[6]

We will take Characteristic equation of equations (E_4_d)

$K_{q+1} - 3\alpha K_q = 0$

$r^{q+1} - 3\alpha r^q = 0$

$r - 3\alpha = 0$

$r = 3\alpha$

$K_q = A3^q\alpha^q$

We will take Characteristic equation of equations (E_5_e)

$Y_{t+1} - \alpha Y_t = 0$

$r^{t+1} - \alpha r^t = 0$

$r - \alpha = 0$

$r = \alpha$

$Y_t = B\alpha^t$

Then $S(q,t) = K_q Y_t$

$= A3^q\alpha^q B\alpha^t$

$= \boldsymbol{AB\ 3^q\alpha^{q+t}}$

Where A,B are arbitrary constants.

Summing over α allows us to obtain the general solution

$S(q,t) = AB3^q f(q+t).$

Where f is an arbitrary function of $q+t$.

Example 3: find the solution of the partial difference equation by Characteristic equation

$S(q,t) + 4S(q-1,t-1) = 0$

Solution:

To solve this type of equation, we first transform the partial difference equation into a one-variable linear difference recurrence relation, and then apply its characteristic equation.

$$S(q,t) + 4S(q-1,t-1) = 0 \qquad (E_1_a)$$

We will use the Laplace method to convert the partial difference equation (E_1_a) into a linear difference equation through:

Let $q - t = g$. There for, setting

$$X(q) = S(q, q - g) \qquad (E_2_b)$$

By substituting Equation (E.2.b) into equation (E.1.a) yields;

$$X(q) + 4X(q-1) = 0 \qquad (E_3_c)$$

the equations (E_3_c) is linear difference equations[6]

We will take Characteristic equation of equations (E_3_c)

$X(q) + 4X(q-1) = 0$

$r^{q-1} + 4r^{q} = 0$

$r + 4 = 0$

$r = -4$

$\mathrm{X(q)} = \mathrm{C}(-4)^{\mathrm{q}}$

If we now replace g by $q - t$ and C by $F(q-t)$, we obtain

$S(q,t) = C(-4)^{q} F(q-t).$

where C is an arbitrary constant and F is an arbitrary function determined by the boundary or initial conditions.

Conclusions and Recommendations

First: Conclusions

1- Linear partial difference equations cannot be solved directly using the characteristic equation method unless they are first transformed into linear difference equations using one of the transformation methods.

2- We have found that the highly characteristic equation method is a very easy and simple way to solve this type of equation.

Second: Recommendations

1- It is recommended to use the characteristic equation method to solve a system of nonlinear difference equations

2- I recommend using the characteristic equation method to solve the system of partial difference equations

Conclusion

This paper deals with the solution of partial difference equations by the method of characteristic equations. In fact, the problems of partial difference equations cannot be directly solved by using the method of characteristic equations. So, we give the methods of transforming the partial difference equations into problems of linear difference equations which can be directly solved by the method of characteristic equations.

Conflict of Interest

The authors declare that they have no conflicts of interest.

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